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BAYESIAN SINGLE SAMPLING ATTRIBUTE PLANS FOR DISCRETE PRIOR DISTRIBUTIONS

BY

A. HALD

København 1965

Kommissionær: Ejnar Munksgaard

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Synopsis

The paper gives a rather complete tabulation and discussion of properties of a system of single sampling attribute plans obtained by minimizing average costs under the assumptions that costs are linear in p, the fraction defective, and that the distribution of lot quality is a double binomial distribution. The optimum sampling plan (n,c) depends on 6 parameters $(N, p_r, p_s, p_1, p_2, w_2)$ where N denotes lot size, (p_r, p_s) are suitably normalized cost parameters, and (p_1, p_2, w_2) are the parameters of the prior distribution. It may be shown, however, that the weights combine with the p's in such a way that only 5 independent parameters are left.

A procedure to obtain the exact solution of the problem has been developed in a previous paper and this procedure is used for computing a set of *master tables* in which $p_r = p_s = 0.01$ and 0.10, $w_2 = 0.05$, (p_1, p_2) take on suitably chosen values in relation to the value of p_r, and $1 \leq N \leq 200{,}000$.

The properties of the optimum plans are studied, and *simple conversion formulas* are derived which makes it possible to find the optimum plan for an arbitrary set of parameters from a plan in the master tables with a "corresponding" set of parameters. The main tool for this investigation is *the asymptotic expressions for the acceptance number and for the sample size*, viz. $c = np_0 + a + o(1)$ and $n = \frac{1}{\varphi_0}\left(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_0\right) + o(1)$, where p_0 and φ_0 are functions of (p_1, p_2) only, whereas a and λ depend on the other parameters also. It is furthermore shown that the minimum value of the standardized costs per lot asymptotically equals the costs of sampling inspection plus a constant and that the producer's and the consumer's risks tend to zero inversely proportional to lot size. By means of the asymptotic formulas it is possible to find out how (n,c) vary with the individual parameters and derive two general conversion formulas.

Efficiency of various other systems of sampling plans is studied in relation to the present model and some general recommendations are made.

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CONTENTS

1. Introduction and Summary

The main purpose of the present paper is to give a rather complete tabulation and discussion of properties of a system of single sampling attribute plans obtained by minimizing average costs under the assumptions that costs are linear in p, the fraction defective, and that the distribution of lot quality is a double binomial distribution.

Starting from a cost function containing 6 parameters and a mixed binomial prior distribution it is shown how the *average* costs may be written in a *standard form* containing only *two parameters*, p_r and p_s, besides the parameters defining the prior distribution. The one parameter, p_r, is the economic break-even quality and depends on the costs of acceptance and rejection only, whereas the second parameter, p_s, also depends on the costs of sampling inspection and the average quality. In a simple and practically important case p_r and p_s denote the costs of rejection and the costs of sampling inspection, respectively, divided by the costs of accepting a defective item.

Specializing the prior distribution to a *double binomial distribution* defined by the two quality levels (p_1,p_2) and the weights (w_1,w_2), $w_1 + w_2 = 1$, it will be seen that the optimum sampling plan (n,c) depends on the 6 parameters (N,p_r,p_s,p_1,p_2,w_2) where N denotes lot size. It may be shown, however, that the weights combine with the p's in such a way that only 5 (independent) parameters are left.

A procedure to obtain the exact solution of the problem has been developed in a previous paper and this has been used for computing a set of *master tables* in which $p_r = p_s = 0.01$ and 0.10, $w_2 = 0.05$, (p_1,p_2) take on suitably chosen values in relation to the value of p_r, and $1 \leqq N \leqq 200{,}000$.

In the remaining part of the paper the properties of the optimum plans are studied with the purpose to derive *simple conversion formulas* which will make it possible to find the optimum plan for an arbitrary set of parameters from a plan in the master table with a "corresponding" set of parameters. The main tool for this investigation is *the asymptotic expressions for the acceptance number and for the sample size*, viz.

$$c = np_0 + a + o(1) \quad \text{and} \quad n = \frac{1}{\varphi_0}\left(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_0\right) + o(1),$$

where p_0 and φ_0 are functions of (p_1,p_2) only, whereas a and λ depend on the other parameters also. It is furthermore shown that the minimum value of the standardized costs per lot asymptotically equals the costs of sampling inspection plus a constant (depending on (p_1,p_2)) and that the producer's and the consumer's risks tend to zero inversely proportional to lot size. Numerical investigations show that the asymptotic expressions give good approximations to the optimum plan even for quite small values of N.

By means of the asymptotic formulas it is possible to find out how (n,c) vary with the individual parameters. One of the most important results is found by letting all the p's tend to zero which leads to "*the proportionality law*": The optimum sampling plan corresponding to $(N,\lambda p_r,\lambda p_s,\lambda p_1,\lambda p_2,w_2)$ is approximately equal to $(n^*/\lambda,c^*)$ where (n^*,c^*) is the plan corresponding to $(N^*,p_r,p_s,p_1,p_2,w_2)$ with $N^* = N\lambda$.

This theorem combined with other similar results regarding the effect of varying the individual parameters lead to *two general conversion formulas* stated in sections 8 and 11. A summary of these formulas is given at the end of the paper in connection with the tables.

Efficiency of a sampling plan is defined as the ratio of the standardized costs (loss) of the optimum plan and the costs of the plan in question. Efficiency is discussed for various alternative systems and the efficiency of using optimum plans determined from wrong values of the parameters is studied.

Finally the present system is discussed in relation to other systems and it is pointed out that *from an economic point of view it is not advisable to fix the consumer's or the producer's risk.* If one wants a system with a fixed risk then *the risk should be fixed to 50 per cent at a point between p_1 and p_2*. Two such IQL systems are then briefly discussed.

2. The model

Several authors have studied economic models, mostly linear, for the determination of single sampling inspection plans by attributes, see for instance [1] and [2].

We shall here start from the formulation proposed by Guthrie and Johns [3] and show how the model may be reduced to a *standard form* as previously used by Hald [4].

Let N and n denote lot size and sample size and let X and x denote number of defectives in the lot and the sample, respectively. The acceptance number is denoted by c.

Let the costs be

$$nS_1 + xS_2 + (N-n)A_1 + (X-x)A_2 \quad \text{for } x \leqq c \tag{1}$$

and

$$nS_1 + xS_2 + (N-n)R_1 + (X-x)R_2 \quad \text{for } x > c. \tag{2}$$

The interpretation of the six cost parameters depends on the kind of inspection envisaged, i.e. whether inspection is a consumer's receiving inspection, a producer's inspection of finished goods, or "internal inspection" by delivery of goods from one department to another within the same firm. The cost parameters may have quite different values when considered exclusively from a producer's or a consumer's point of view because certain costs are borne primarily by one of the parties involved. The values of the cost parameters also depend on whether the inspection is rectifying or non-rectifying, destructive or non-destructive. In the following the two cost expressions are discussed and a few *examples* of interpretation are given.

Costs associated with the sample, $nS_1 + xS_2$, for brevity called "*costs of sampling inspection*" consist of two parts: one part, nS_1, proportional to the number of items in the sample so that S_1 includes *sampling and testing costs per item,* and another part, xS_2, proportional to the number of defectives in the sample, i.e. S_2 denotes *additional costs for an inspected defective item.* If defective items found in the sample are repaired, say, then S_2 includes the repair costs per item.

"*Costs of acceptance*" are similarly composed of a part, $(N\text{-}n)A_1$, proportional to *the number of items in the remainder of the lot,* and another part, $(X\text{-}x)A_2$, proportional to *the number of defective items accepted.* Whereas A_1 usually will be zero or negligible, A_2 will often be considerable. If accepted items are used as parts in an assembly operation, say, A_2 may include the manufacturing costs (or the price) of an item, the costs of handling the defective item in assembling and disassembling, and the damage to other parts used in the assembly. In case of inspection of finished goods A_2 may include costs of repair, service and guarantees plus loss of good-will.

"*Costs of rejection*" consist of a part, $(N\text{-}n)R_1$, proportional to *the number of items in the remainder of the lot,* and another part, $(X\text{-}x)R_2$, proportional to *the number of defective items rejected. Rejection* is here taken in a broad sense meaning only that the lot cannot be accepted according to the sampling plan used. Rejection may therefore lead to sorting, price reduction, scrapping, or salvaging. If rejection means sorting, say, then R_1 includes sorting costs per item and R_2 denotes *additional costs for defective items found,* for example costs of repair or replacement.

It is obvious that from a practical point of view it will in general be easiest to obtain information on the values of the cost parameters in the case of "internal inspection".

Denoting the hypergeometric probability by

$$p\{x\,|\,X\} = \binom{n}{x}\binom{N-n}{X-x}\Big/\binom{N}{X}$$

the average costs for lots of size N with X defectives become

$$\left.\begin{aligned} K(N,n,c,X) = \sum_{x=0}^{n}(nS_1 + xS_2)p\{x\,|\,X\} + \sum_{x=0}^{c}\big((N-n)A_1 + (X-x)A_2\big)p\{x\,|\,X\} \\ + \sum_{x=c+1}^{n}\big((N-n)R_1 + (X-x)R_2\big)p\{x\,|\,X\}. \end{aligned}\right\} \qquad (3)$$

Let $f_N(X)$ denote the (prior) distribution of X, i.e. the distribution of *lot* quality. The average costs then become

$$K(N,n,c) = \sum_X K(N,n,c,X) f_N(X). \tag{4}$$

As shown in [4] this expression becomes *linear in N* for the important class of *mixed binomial distributions,* i.e. for

$$f_N(X) = \int_0^1 \binom{N}{X} p^X q^{N-X} dW(p) \tag{5}$$

where $W(p)$ denotes a cumulative distribution function (independent of N).

From (3)–(5) we find

$$K(N,n,c) = \int_0^1 K(N,n,c,p) dW(p) \tag{6}$$

where

$$K(N,n,c,p) = n(S_1 + S_2 p) + (N-n)\big((A_1 + A_2 p)P(p) + (R_1 + R_2 p)Q(p)\big), \tag{7}$$

$$P(p) = B(c,n,p) = \sum_{x=0}^{c} \binom{n}{x} p^x q^{n-x}, \tag{8}$$

and $Q(p) = 1 - P(p)$.

For convenience the frequency function corresponding to $W(p)$ will be called the *distribution of the process average* or the *distribution of p* as distinct from $f_N(X)$ which gives the distribution of X/N, i.e. *the distribution of lot quality.* (The following discussion will be in terms of p).

Limiting the prior distributions to mixed binomials, (6) *shows that the average costs may be considered as an average of the cost function* (7), *which is a function of p, with respect to the distribution of p*. It should be noted that this result is valid for any (N,n) for a mixed binomial prior distribution and that a similar result holds for $N \to \infty$, $n \to \infty$, and $n/N \to 0$, for any prior distribution. The limit theorems derived in the following may therefore be applied in general.

The sampling plans discussed are obtained by minimizing $K(N,n,c)$ according to (6) with respect to (n,c) for given cost parameters and prior distribution and they will be called Bayesian single sampling plans or optimum plans.

Starting from (7) we introduce the three cost functions

$$k_s(p) = S_1 + S_2 p, \tag{9}$$

$$k_a(p) = A_1 + A_2 p, \tag{10}$$

and

$$k_r(p) = R_1 + R_2 p, \tag{11}$$

defined for $0 \leqq p \leqq 1$. We shall make the following assumptions regarding these functions:

1. All three functions are non-negative and none of them is identical zero.
2. $k_a(0) < k_r(0)$ and $k_a(1) > k_r(1)$, from which follows that the equation $k_a(p) = k_r(p)$ has the solution

$$p_r = (R_1 - A_1)/(A_2 - R_2), \quad 0 < p_r < 1, \tag{12}$$

p_r being called the (economic) *break-even quality*.

3. $k_s(p) \geqq k_m(p)$ for $0 \leqq p \leqq 1$, where

$$k_m(p) = \begin{cases} k_a(p) & \text{for } p \leqq p_r \\ k_r(p) & \text{for } p > p_r. \end{cases} \tag{13}$$

The function $k_m(p)$ gives the unavoidable (minimum) costs, i.e. the costs corresponding to the situation where perfect knowledge of quality exists *without costs* and *all lots are classified correctly* on basis of the corresponding process average, viz. accepted for $p \leqq p_r$ and rejected for $p > p_r$.

Averages over the prior distribution are denoted by k_s, k_a, etc., i.e.

$$k_a = \int_0^1 k_a(p)dW(p) = k_a(\bar{p}) = A_1 + A_2\bar{p}, \tag{14}$$

and

$$k_m = \int_0^1 k_m(p)dW(p) = \int_0^{p_r} k_a(p)dW(p) + \int_{p_r}^1 k_r(p)dW(p). \tag{15}$$

Costs per *item* are denoted by k, costs per *lot* by the corresponding K, i.e. $K = Nk$.

The average costs for the *three cases without sampling inspection*, i.e. the cases where

(a) all lots are classified correctly,
(b) all lots are accepted, and
(c) all lots are rejected,

then become k_m, k_a, and k_r, respectively. These cases are useful "reference cases" since sampling inspection is justified only if $k - k_m < \min\{k_a - k_m, k_r - k_m\}$, where $k = K(N,n,c)/N$.

Case (a) will usually be considered as the basic reference case and average costs for other cases will therefore be reduced by k_m, since k_m represents the average fixed costs per item which will be incurred irrespective of the decision made. The *cost differences*

$$k_a - k_m = \int_{p_r}^{1} (k_a(p) - k_r(p))dW(p)$$

and

$$k_r - k_m = \int_{0}^{p_r} (k_r(p) - k_a(p))dW(p)$$

represent *average decision losses* in case (b) and (c) respectively, and $k_s - k_m$ represents the average "loss" by inspection.

From (6) and (15) we find

$$K = nk_s + (N-n)\int_0^1 (k_a(p)P(p) + k_r(p)Q(p))dW(p) \tag{16}$$

and

$$K_m = nk_m + (N-n)\left\{\int_0^{p_r} k_a(p)dW(p) + \int_{p_r}^1 k_r(p)dW(p)\right\}$$

leading to

$$K - K_m = n(k_s - k_m)$$
$$+ (N-n)\left\{\int_0^{p_r} (k_r(p) - k_a(p))Q(p)dW(p) + \int_{p_r}^1 (k_a(p) - k_r(p))P(p)dW(p)\right\}$$

$$= n(k_s - k_m) + (N-n)(A_2 - R_2)\left\{\int_0^{p_r} (p_r - p)Q(p)dW(p) + \int_{p_r}^1 (p - p_r)P(p)dW(p)\right\}, \tag{17}$$

the two terms giving the average costs of sampling inspection and the average decision losses, respectively.

Instead of minimizing K with respect to (n,c) we might just as well minimize $K - K_m, (K - K_m)/(A_2 - R_2)$, or $(K - K_m)/(k_s - k_m)$, since $K_m, A_2 - R_2$, and $k_s - k_m$ are independent of (n,c). It will be seen from (17) that it is practical to use $A_2 - R_2$ or $k_s - k_m$ as "economic unit".

Defining

$$p_m = \int_0^{p_r} pdW(p) + \int_{p_r}^1 p_r dW(p) = p_r - \int_0^{p_r} (p_r - p)dW(p), \tag{18}$$

we find $0 \leqq p_m \leqq p_r$ and

$$p_r - p_m = (k_r - k_m)/(A_2 - R_2). \tag{19}$$

Defining p_s by means of

$$p_s - p_m = (k_s - k_m)/(A_2 - R_2) \tag{20}$$

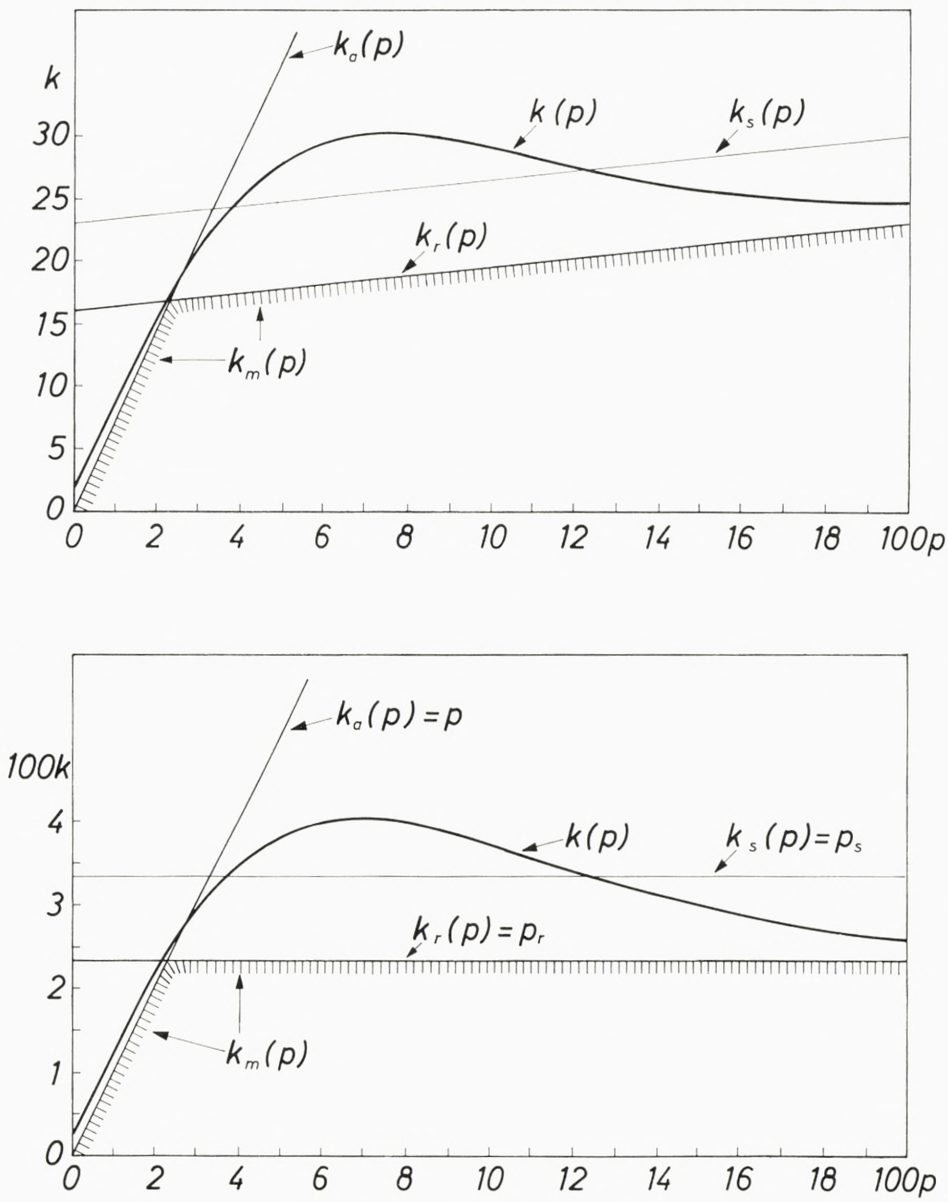

Fig. 1. Example of cost functions.

we find

$$p_s - p_r = (k_s - k_r)/(A_2 - R_2) \tag{21}$$

and

$$p_s = \{(S_1 - A_1) + (S_2 - R_2)\bar{p}\}/(A_2 - R_2). \tag{22}$$

Introducing

$$R^*(N,n,c) = \{K(N,n,c) - K_m\}/(A_2 - R_2) \tag{23}$$

we find the *standard form*

$$R^* = n(p_s - p_m) + (N-n)\left\{\int_0^{p_r}(p_r - p)Q(p)dW(p) + \int_{p_r}^{1}(p - p_r)P(p)dW(p)\right\} \tag{24}$$

containing only two parameters p_r and $p_s - p_m$, instead of the six cost parameters in the original model, see [4]. It should be noted that $p_s - p_m$ depends on the prior distribution besides on the cost parameters.

Consider the *special case* given by $k_a(p) = A_2 p$, $k_r(p) = R_1$, and $k_s(p) = S_1$, which is a model commonly used in practice. It follows that $p_r = R_1/A_2$ and $p_s = S_1/A_2$, i.e. p_r and p_s are the costs of rejection and of sampling and testing, respectively, measured with the cost of accepting a defective item. This simple interpretation of p_r and p_s is one of the reasons for using them as parameters.

It is often useful to discuss the problem in terms of the simple cost functions $k_a(p) = p$, $k_r(p) = p_r$, and $k_s(p) = p_s$, which immediately lead to the form (24). The corresponding form of (7) becomes

$$K_0(p) = np_s + (N-n)\big(pP(p) + p_r Q(p)\big)$$

from which the general form may be found as

$$K(p) = (A_2 - R_2)K_0(p) + \big(nS_2 + (N-n)R_2\big)p + \big(NA_1 - n(S_2 - R_2)\bar{p}\big).$$

A sketch of the cost functions for a typical case has been given in Fig. 1, which is based on the data in section 13.

For some purposes it is useful to use $k_s - k_m$ as economic unit instead of $A_2 - R_2$. Putting

$$R(N,n,c) = \{K(N,n,c) - K_m\}/(k_s - k_m),$$

i.e.

$$R = R^*/(p_s - p_m),$$

we find

$$R = n + \frac{N-n}{p_s - p_m}\left\{\int_0^{p_r}(p_r - p)Q(p)dW(p) + \int_{p_r}^{1}(p - p_r)P(p)dW(p)\right\}, \tag{25}$$

the two terms again giving the costs of sampling inspection and the average decision losses, respectively, but here using the average costs of sampling inspection (minus k_m) per item in the sample as economic unit.

In the next section we shall discuss the determination of (n,c) for a *double binomial distribution* as prior distribution. This means that p is a random variable taking on only two values, $p_1 < p_r < p_2$, with probabilities w_1 and $w_2 = 1 - w_1$, respectively. From (25) we then find

$$R = n + (N-n)\big(\gamma_1 Q(p_1) + \gamma_2 P(p_2)\big) \tag{26}$$

where

$$\gamma_i = |p_i - p_r| w_i/(p_s - p_m) = |k_a(p_i) - k_r(p_i)| w_i/(k_s - k_m), \quad i = 1,2, \tag{27}$$

$$p_m = p_1 w_1 + p_r w_2, \tag{28}$$

i.e. R depends on four parameters only, viz. $p_1, p_2, \gamma_1, \gamma_2$.

The correspondingly standardized costs for the cases of acceptance and rejection without inspection are

$$R_a = N(k_a - k_m)/(k_s - k_m) = N\gamma_2 \tag{29}$$

and

$$R_r = N(k_r - k_m)/(k_s - k_m) = N\gamma_1. \tag{30}$$

These results may also be obtained from (26) for $n = 0$ by setting $P(p) = 1$ and 0, respectively.

If acceptance without inspection is cheaper than rejection without inspection, i.e. $k_a < k_r$ we find $\bar{p} < p_r$ and $\gamma_2 < \gamma_1$.

In the special case $k_s = k_r$ we have $p_s = p_r$ and $\gamma_1 = 1$ so that the model contains only three parameters.

It should be noted that

$$\gamma_2 = \frac{\bar{p} - p_m}{p_s - p_m} = 1 - \frac{p_s - \bar{p}}{p_s - p_m} \tag{31}$$

and

$$\gamma_1 = \frac{p_r - p_m}{p_s - p_m} = 1 - \frac{p_s - p_r}{p_s - p_m}. \tag{32}$$

3. The exact solution and the tables

In a previous paper [4] we have proved the following *theorem*:

For a double binomial (prior) distribution of lot quality given by the parameters (p_1, p_2, w_2) *and for linear cost functions* (1) *and* (2) *the Bayesian single sampling plan may be found by minimizing* $R(N,n,c)$, *see* (26), *with respect to* (n,c). *The solution satisfies the two inequalities*

$$\alpha + \beta c \leqq n < \alpha + \beta(c+1) \tag{33}$$

and

$$F(n-1, c) \leqq N < F(n,c) \tag{34}$$

where

$$\alpha = \log \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)} \Big/ \log \frac{q_1}{q_2} = \log \frac{\gamma_2}{\gamma_1} \Big/ \log \frac{q_1}{q_2}, \tag{35}$$

$$\beta = \log \frac{p_2 q_1}{q_2 p_1} \Big/ \log \frac{q_1}{q_2}, \tag{36}$$

and

$$F(n,c) = n + 1 + \frac{p_s - p_r + \sum_i w_i(p_r - p_i)B(c,n,p_i)}{\sum_i w_i(p_i - p_r)p_i b(c,n,p_i)}. \tag{37}$$

For two plans (n_1, c_1) *and* (n_2, c_2), $c_1 < c_2$ *say, satisfying* (33) *and having overlapping N-intervals according to* (34) $R(N, n_1, c_1) \lesseqgtr R(N, n_2, c_2)$ *for* $N \lesseqgtr N_{12}$ *where*

$$N_{12} = \frac{(p_s - p_r)(n_2 - n_1) + n_2\gamma(n_2, c_2) - n_1\gamma(n_1, c_1)}{\gamma(n_2, c_2) - \gamma(n_1, c_1)} \tag{38}$$

and

$$\gamma(n,c) = \sum_i w_i(p_r - p_i)B(c,n,p_i). \tag{39}$$

In [4] the theorem was derived as a special case of a more general one. We shall here derive the theorem directly from (26) using the same method as in [4].

Values of (n,c) minimizing R must satisfy the two inequalities

$$\Delta_c R(N,n,c-1) \leqq 0 < \Delta_c R(N,n,c), \qquad 0 \leqq c \leqq n, \tag{40}$$

and

$$\Delta_n R(N,n-1,c) \leqq 0 < \Delta_n R(N,n,c), \qquad c \leqq n \leqq N, \tag{41}$$

Δ denoting the usual forward difference operator.

Noting that $\Delta_c B(c,n,p) = b(c+1,n,p)$ and $\Delta_n B(c,n,p) = -pb(c,n,p)$ we find from (26)

$$\Delta_c R(N,n,c) = (N-n)\{-\gamma_1 b(c+1,n,p_1) + \gamma_2 b(c+1,n,p_2)\} \tag{42}$$

and

$$\Delta_n R(N,n,c) = 1 - \{\gamma_1 Q(p_1) + \gamma_2 P(p_2)\} + (N-n-1)\{\gamma_1 p_1 b(c,n,p_1) - \gamma_2 p_2 b(c,n,p_2)\}. \tag{43}$$

Inserting these expressions into (40) and (41) and solving for n and N, respectively, immediately leads to (33) and (34). From $R(N,n_1,c_1) = R(N,n_2,c_2)$ we next determine N_{12} by solving for N.

A sketch of R as function of n and c for fixed N has been given in Fig. 2 for a typical case.

The economic interpretation of (40) and (42) is the following: For given n the optimum value of c is determined such that a change of c, an increase by 1 say, will give nearly no change of the total decision loss, since the loss due to the increased consumer's risk is nearly balanced by the gain due to the smaller producer's risk.

Similarly the interpretation of (41) and (43) is that for given c the optimum value of n is determined such that a change of n, an increase by 1 say, will give nearly no change of total costs, since the increase of sampling inspection costs by 1 minus the average decision loss for one item is nearly balanced by the decrease in decision losses for the remainder of the lot.

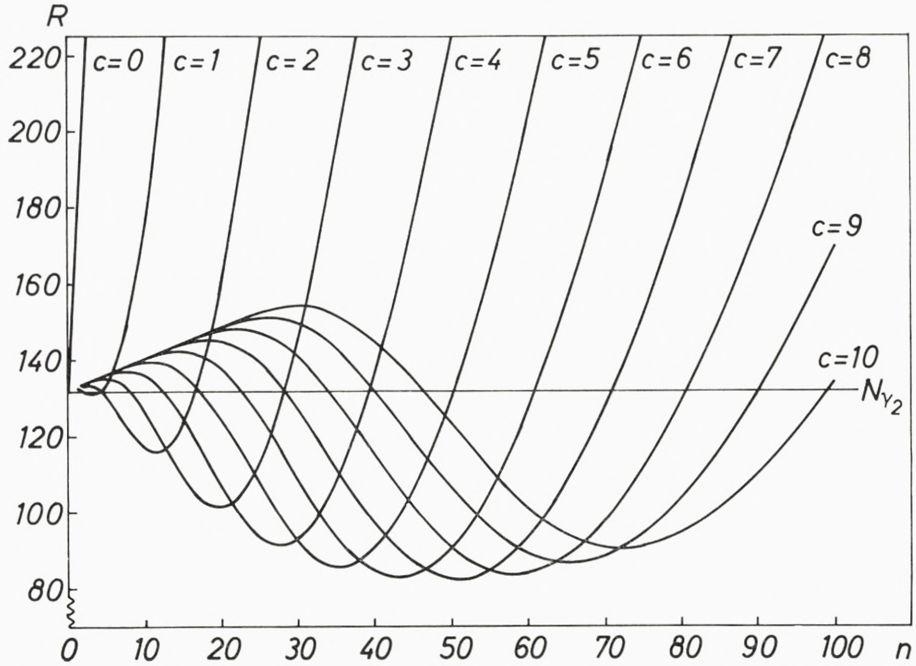

Fig. 2. $R(N,n,c)$ as function of n and c for $N = 1000$, $p_r = p_s = 0.10$, $p_1 = 0.06$, $p_2 = 0.20$, and $w_2 = 0.05$.

Tabulation of optimum plans may be carried out by starting from the smallest value of c giving a positive $n\,(n \geqq c)$ according to (33), i.e. $c_m = [-\alpha/(\beta-1)]$, [ ] denoting "the integer part of". For consecutive values of c, n- and N-intervals are computed from (33) and (34) and in case of overlapping N-intervals costs are compared by means of (38). A detailed example may be found in [4]. The tables have been computed by this method on an electronic computer.

The sampling plans have been tabulated for two "quality levels", viz. $p_r = p_s = 0.01$ and 0.10, for one value of the weight function $w_2 = 0.05$, for 8 values of p_1/p_r, and for 10, respectively 5, values of p_2/p_r, giving a total of 120 tables. Each table gives (n,c) as function of N for $N \leqq 200{,}000$.

For $p_r = 0.01$ the search for optimum plans has been limited to values of n which are multiples of 5.

These tables will be referred to as "*master tables*" since *optimum plans for other values of the parameters may easily be found from the tabulated ones* by means of conversion formulas developed in the following sections.

The exact solution has been modified in one respect. For *a given value of c* the first and last N-interval may be rather short as compared to the other intervals. As an example consider the following section of the original table for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.020$, $w_2 = 0.05$:

N	n	c	ΔN
4010–4370	165	3	360
4370–4420	170	3	50
4420–4430	240	4	10
4430–4920	245	4	490
4920–5570	250	4	650
5570–5590	255	4	20
5590–5610	325	5	20
5610–6250	330	5	640

The example shown is an extreme one with small intervals occurring at the beginning as well as at the end of each section of the table. It is naturally without any interest to use the sampling plan (240,4) for $4420 < N < 4430$ and then change to (245,4) for $4430 < N < 4920$. To eliminate such small intervals from the final table it was decided to discard the first and the last sampling plan for a given c if the length of the corresponding N-interval was less than 1/5 of the length of the neighbouring interval. In such cases the value of N according to (38) was computed for the new neighbouring plans, (165,3) and (245,4) say, to find the optimum N-intervals for the remaining plans. The result of this procedure is in most cases practically equal to incorporating the small N-intervals into the larger neighbouring intervals, for example using (245,4) for $4420 < N < 4920$.

To save space every second N-interval for *a given value of* c has been omitted because *the corresponding sampling plans may be found by adding* 1 ($p_r = 0.10$) *and* 5 ($p_r = 0.01$), *respectively, to* n *for the preceding interval.*

Values of N have been rounded to 3 significant figures and tabulation has been stopped at $N = 200{,}000$.

As mentioned above the tables were designed as master tables from which optimum plans may be derived for other values of the parameters and for this reason it was decided to tabulate the complete solution with respect to N to make interpolation superfluous.

The user of the tables in practice may easily derive a simplified set of tables from the given ones, either by using a set of fixed N-intervals, or a set of fixed N-arguments. An example has been given in the table on page 17.

The "natural" parameters of the model are (p_1, p_2, w_2), which characterize the prior distribution, and (p_r, p_s), which depend on the costs. The tables and the properties of the solution will be discussed in terms of these parameters on basis of the results in the next section. However, one property may be stated immediately from the observation that the solution depends on four parameters only, viz. $(p_1, p_2, \gamma_1, \gamma_2)$. The three parameters (p_r, p_s, w_2) may therefore in respect to the solution be considered as functionally related, i.e. *combinations of* (p_r, p_s, w_2) *giving the same* (γ_1, γ_2) *will lead to the same sampling plan.*

Single Sampling Tables for $100p_r = 100p_s = 1.0$, $100p_1 = 0.5$, and $w_2 = 0.05$.

$100p_2$	1.5		2.0		3.0		4.0		5.0		6.0		7.0	
N	n	c	n	c	n	c	n	c	n	c	n	c	n	c
20											Accept		5	0
30									Accept		5	0	5	0
50									5	0	10	0	10	0
70									5	0	10	0	15	0
100							Accept		10	0	15	0	15	0
200							10	0	20	0	20	0	25	0
300							15	0	50	1	50	1	50	1
500					Accept		55	1	60	1	60	1	55	1
700					45	1	65	1	65	1	65	1	60	1
1000					55	1	110	2	105	2	100	2	90	2
2000					125	2	170	3	155	3	140	3	100	2
3000			Accept		195	3	180	3	160	3	145	3	135	3
5000			185	3	265	4	235	4	210	4	185	4	140	3
7000			275	4	330	5	290	5	215	4	190	4	170	4
10000			450	6	400	6	295	5	260	5	195	4	175	4
20000	Accept		640	8	475	7	355	6	310	6	240	5	215	5
30000	755	9	735	9	545	8	410	7	315	6	280	6	220	5
50000	1080	12	920	11	620	9	470	8	365	7	285	6	255	6
70000	1300	14	1095	13	690	10	475	8	410	8	325	7	260	6
100000	1520	16	1190	14	760	11	530	9	415	8	330	7	290	7

From

$$\frac{\gamma_2}{\gamma_1} = \frac{(p_2 - p_r)w_2}{(p_r - p_1)w_1}$$

we find

$$p_r - p_1 = (p_2 - p_1)\Big/\left(1 + \frac{\gamma_2 w_1}{\gamma_1 w_2}\right). \tag{44}$$

From

$$\gamma_1(p_s - p_m) = (p_r - p_1)w_1$$

and

$$p_s - p_m = p_s - p_1 - w_2(p_r - p_1)$$

we find

$$p_s - p_1 = (p_r - p_1)\left(\frac{w_1}{\gamma_1} + w_2\right). \tag{45}$$

These formulas show how p_r and p_s depend on w_2 for given $(p_1, p_2, \gamma_1, \gamma_2)$. To use them in connection with the master tables we put $p_r = p_s$ and $w_2 = 0.05\lambda$ which leads to

$$p_r(\lambda) = p_{10} + (p_{20} - p_{10}) \Big/ \left(1 - \gamma_{20} + \frac{20\gamma_{20}}{\lambda}\right)$$

where

$$\gamma_{20} = \frac{p_{20} - p_{r0}}{19(p_{r0} - p_{10})} = \frac{\varrho_2 - 1}{19(1 - \varrho_1)},$$

the index 0 denoting an argument in the master table, $p_{r0} = 0.01$ or 0.10, $\varrho_i = p_{i0}/p_{r0}$. Dividing by p_{r0} gives

$$p_r(\lambda)/p_{r0} = \varrho_1 + (\varrho_2 - \varrho_1) \Big/ \left(1 - \gamma_{20} + \frac{20\gamma_{20}}{\lambda}\right) = f(w_2, \varrho_1, \varrho_2) \tag{46}$$

which has been tabulated in the appendix.

The field of application of the master tables may therefore be considerably enlarged by making use of the following rule:

The optimum sampling plan for $(N, p_{r0}, p_{10}, p_{20}, w_2 = 0.05)$, $p_{r0} = p_{s0}$, *is the same as the plan for* $(N, p_{r0} f(w_2, \varrho_1, \varrho_2), p_{10}, p_{20}, w_2)$.

Consider for example the case with $p_{r0} = p_{s0} = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, and $w_2 = 0.05$ for which the optimum plans have been given in the master table. The same plans are also optimum for $w_2 = 0.20$, say, and $p_r = p_s = 0.019$, $p_1 = 0.006$, and $p_2 = 0.040$ which may be seen by interpolation in the table of $f(w_2, \varrho_1, \varrho_2)$ for $\varrho_1 = 0.6$ and $\varrho_2 = 4.0$.

4. The asymptotic solution

In this section we shall give a somewhat simpler and more direct proof of the asymptotic results found by Guthrie and Johns [3] and by Hald [4], and furthermore carry the asymptotic expansion so far that we get a useful approximation to the exact solution also for small values of c.

The proof is based on the following lemma which is a special case of a theorem proved by Blackwell and Hodges [5]:

For $c/n = h = p_0 + \varepsilon$, p_0 being a constant and $\varepsilon \to 0$ for $n \to \infty$, we have

$$P(p) = \frac{1}{\sqrt{2\pi n p_0 q_0}} \frac{q_0 p}{(p - p_0)} e^{-n\varphi(h,p)} \left(1 + O(\sqrt{\varepsilon})\right) \quad \text{for } p_0 < p, \tag{47}$$

where

$$\varphi(h,p) = h \ln \frac{h}{p} + (1 - h) \ln \frac{1 - h}{1 - p}. \tag{48}$$

For $p_0 > p$ the same expression is valid for $Q(p)$ if only $p - p_0$ is replaced by $p_0 - p$.

Writing

$$\varphi(h,p) = \varphi(p_0,p) + \varepsilon\varphi'(p_0,p) + O(\varepsilon^2) \tag{49}$$

where

$$\varphi'(p_0,p) = \ln\frac{p_0 q}{q_0 p} \tag{50}$$

we find from (26) and (47) the asymptotic expression

$$R = n + (N-n)\frac{q_0}{\sqrt{2\pi n p_0 q_0}}\sum_{i=1}^{2}\frac{\gamma_i p_i}{|p_0 - p_i|}e^{-n\varphi(p_0,\,p_i) - n\varepsilon\varphi'(p_0,\,p_i)}\left(1 + O(\sqrt{\varepsilon})\right) \tag{51}$$

on the assumption that $p_1 < p_0 < p_2$. (As will be shown later $\varepsilon = O(1/n)$, and we may therefore disregard $n\varepsilon^2$). We shall first determine the value of $h = p_0 + \varepsilon$ which minimize R for given n and next determine the value of n giving the absolute minimum by treating R as a differentiable function of n.

The essential feature of (51) is that the two binomial risks, $Q(p_1)$ and $P(p_2)$, have been expressed as functions tending exponentially to zero for $n \to \infty$.

As explained in [4] the optimum plan must have the property that $R/N \to 0$ for $N \to \infty$, $n \to \infty$, and $n/N \to 0$. It follows that p_0 must satisfy the inequality $p_1 < p_0 < p_2$ because otherwise R/N would not tend to zero but to γ_1 or γ_2.

We shall state the theorem to be proved for the double binomial distribution only, but it is valid for a more general class of distributions, viz. for a distribution having probability density $w(p) = 0$ for $p_1 < p < p_2$, $w(p_1) = w_1 > 0$, $w(p_2) = w_2 > 0$, $w_1 + w_2 \leqq 1$, and

$$\int_0^{p_1^*} dW(p) + \int_{p_2^*}^{1} dW(p) = 1 - w_1 - w_2$$

for $0 \leqq p_1^* < p_1$ and $p_2 < p_2^* \leqq 1$, which means that the probability distribution may be arbitrary outside the interval $p_1^* < p < p_2^*$. The result of such a generalization will only be to add a term to (51) of form

$$\frac{N-n}{p_s - p_m}\,\frac{q_0}{\sqrt{2\pi n p_0 q_0}}\int_I \frac{(p_r - p)p}{p_0 - p}e^{-n\varphi(h,\,p)}dW(p),$$

(I denoting the intervals $(0 \leqq p \leqq p_1^*)$ and $(p_2^* \leqq p \leqq 1)$) which obviously is $O(e^{-n})$ times the last term of (51) since $\varphi(h,p) > \varphi(h,p_1)$ for $p < p_1$ and $\varphi(h,p) > \varphi(h,p_2)$ for $p > p_2$.

Because of the factor $p_r - p$ in the cost function we might also have assumed that $w(p_r) > 0$ without altering the result.

It is reasonable to assume that the two exponential terms in (51) tend to zero with the same speed, i.e. that p_0 is determined from

$$\varphi(p_0, p_1) = \varphi(p_0, p_2)$$

which gives

$$p_0 = \left(\ln \frac{q_1}{q_2}\right) \Big/ \left(\ln \frac{p_2 q_1}{q_2 p_1}\right) = \frac{1}{\beta} \tag{52}$$

and

$$\varphi_0 = p_0 \ln \frac{p_0}{p_i} + q_0 \ln \frac{q_0}{q_i}, \quad i = 1 \text{ or } 2. \tag{53}$$

Under this assumption we shall determine ε by minimization of (51). The part of R depending on ε is

$$f(\varepsilon) = \sum_i \frac{\gamma_i p_i}{|p_0 - p_i|} e^{-n\varepsilon\varphi'(p_0, p_i)}.$$

From $f'(\varepsilon) = 0$ we find

$$\sum_{i=1}^{2} \frac{\gamma_i p_i \varphi_i'}{|p_0 - p_i|} e^{-n\varepsilon\varphi_i'} = 0 \tag{54}$$

where—according to (50)—

$$\varphi_i' = \ln \frac{p_0 q_i}{q_0 p_i}. \tag{55}$$

Solving for $a = n\varepsilon$ we find

$$a\delta_0' = \ln \frac{\gamma_1 p_1 (p_2 - p_0)\varphi_1'}{\gamma_2 p_2 (p_0 - p_1)(-\varphi_2')} \tag{56}$$

where

$$\delta_0' = \varphi_1' - \varphi_2' = \ln \frac{p_2 q_1}{q_2 p_1}. \tag{57}$$

We thus have the result that $c = np_0 + a + o(1)$ in accordance with what could be expected from (33).

Inserting these results into (51) we find

$$R = n + (N - n)\frac{\lambda}{\sqrt{n}} e^{-n\varphi_0} \tag{58}$$

with

$$\lambda = \frac{q_0}{\sqrt{2\pi p_0 q_0}} \sum_{i=1}^{2} \frac{\gamma_i p_i}{|p_0 - p_i|} e^{-a\varphi_i'}. \tag{59}$$

To prove (indirectly) that $h = p_0 + \varepsilon$ minimizes R let us assume that $h = p_0 + \varepsilon$, given by (52) and (56), does not minimize R but that min R is obtained for $h = h_0 + \varepsilon_0$, $h_0 \neq p_0$ and $\varepsilon_0 \to 0$. Denoting the part of R depending on h by $g(h)$ we find for sufficiently large n and for $h_0 < p_0$, say, that

$$g(h_0) = \lambda_1(h_0)e^{-n\varphi(h_0,p_1)}\left(1 + O(e^{-n})\right)$$

since $\varphi(h_0,p_2) > \varphi(h_0,p_1)$ for $h_0 < p_0$. However, $g(h_0)$ cannot be min $g(h)$ since $\varphi(h_0,p_1) < \varphi(p_0,p_1)$, i.e. we have reached a contradiction by assuming $h_0 \neq p_0$.

From $dR/dn = 0$ we find

$$1 - (N-n)\frac{\lambda}{\sqrt{n}}e^{-n\varphi_0}\left(\varphi_0 + \frac{1}{2n}\right) - \frac{\lambda}{\sqrt{n}}e^{-n\varphi_0} = 0 \tag{60}$$

or

$$\ln(N-n) = \varphi_0 n + \frac{1}{2}\ln n - \ln(\lambda\varphi_0) + o(1). \tag{61}$$

From (58) and (60) we also have that

$$\min_{(n,c)} R = n + \frac{1}{\varphi_0} + o(1) \tag{62}$$

where n may be determined by inversion of (61), i.e.

$$n = \frac{1}{\varphi_0}\left(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_0\right) + o(1).$$

We have thus found that asymptotically c is a linear function of n, and n is proportional to $\ln N - \frac{1}{2}\ln\ln N$ plus a constant. Furthermore it follows from (62) that the average decision loss per lot tends to a constant $1/\varphi_0$ so that for large lots decision losses divided by sampling inspection costs tend to zero.

To investigate the two risks asymptotically we find from (54)

$$\frac{\gamma_1 p_1 \varphi_1'}{p_0 - p_1}e^{-a\varphi_1'} = \frac{\gamma_2 p_2(-\varphi_2')}{p_2 - p_0}e^{-a\varphi_2'}$$

so that (59) gives

$$\lambda = \frac{q_0}{\sqrt{2\pi p_0 q_0}}\,\frac{\gamma_1 p_1 \delta_0'}{(p_0 - p_1)(-\varphi_2')}e^{-a\varphi_1'}$$

which together with (60) may be used to reduce

$$Q(p_1) = \frac{1}{\sqrt{2\pi p_0 q_0}}\,\frac{q_0 p_1}{p_0 - p_1}e^{-a\varphi_1'}\frac{1}{\sqrt{n}}e^{-n\varphi_0}$$

to

$$Q(p_1) = \frac{-\varphi_2'}{\varphi_0\gamma_1\delta_0'}\,\frac{1}{N-n}. \tag{63}$$

Similarly we have

$$P(p_2) = \frac{\varphi_1'}{\varphi_0 \gamma_2 \delta_0'} \frac{1}{N-n} \tag{64}$$

so that

$$P(p_2)/Q(p_1) = \gamma_1 \varphi_1' / \gamma_2(-\varphi_2'). \tag{65}$$

We have thus proved the following *theorem*:

Asymptotically the optimum sampling plan is given by

$$c = np_0 + a + o(1) \tag{66}$$

and

$$n = \frac{1}{\varphi_0}\left(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_0\right) + o(1) \tag{67}$$

which lead to

$$\min R = \frac{1}{\varphi_0}\left(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_0 + 1\right) + o(1), \tag{68}$$

$$Q(p_1) = \frac{-\varphi_2'}{\varphi_0 \gamma_1 \delta_0'} \frac{1}{N-n} + o\left(\frac{1}{N}\right)$$

and

$$P(p_2) = \frac{\varphi_1'}{\varphi_0 \gamma_2 \delta_0'} \frac{1}{N-n} + o\left(\frac{1}{N}\right).$$

It will be noted that p_0 and φ_0 depend on (p_1, p_2) only, i.e. they are independent of the cost parameters and w_2.

The asymptotic solution supplements the exact one in several respects. Since the optimum plan is a function of 5 parameters $(N, p_1, p_2, \gamma_1, \gamma_2)$ a complete tabulation is rather hopeless even if a program has been worked out for an electronic computer. Furthermore the properties of the exact solution are not easily to be found from the procedure by which the solution is obtained. The advantages of the asymptotic solution are that

(1) it clearly shows how the optimum plan and various derived quantities depend on the parameters,

(2) it may be used as starting point for developing approximations which are valid also for small N,

(3) it may be used for developing interpolation and extrapolation formulas in connection with "master tables" of the exact solution, and

(4) it shows the sensitivity of the solution with respect to changes of the parameters.

These aspects of the solution will be discussed in the following sections.

5. Comparison of exact and approximate solution

Looking at the relation between n and c in the tables it will be seen that the optimum values of n for a given value of c tend to cluster around

$$n_c = \alpha + \beta\left(c + \frac{1}{2}\right) \tag{69}$$

as might be expected from (33). Comparing with the asymptotic result $c = np_0 + a$, $p_0 = 1/\beta$ and a being defined by (56), agreement between the two expressions would require that

$$\left(\ln \frac{p_2(p_0 - p_1)(-\varphi_2')}{p_1(p_2 - p_0)\varphi_1'}\right) \Big/ \left(\ln \frac{p_2 q_1}{q_2 p_1}\right) = \frac{1}{2}.$$

It can be proved that the ratio on the left hand side above is positive and less that 1. Numerical investigations show that in typical cases in practice the ratio does not deviate much from 1/2. As examples consider the following results:

$100p_1$	$100p_2$	p_2/p_1	Ratio
0.2	4.0	20	0.528
0.2	2.0	10	0.517
0.6	4.0	6.7	0.512
0.6	2.0	3.3	0.505

The ratio depends primarily on p_2/p_1 and practically the same results will be found for values of (p_1, p_2) which are 10 times as large or 1/10 of the values considered. We shall therefore in the following use the simpler expression (69) instead of $c = np_0 + a$ as the starting point for finding n from c or reversely.

The asymptotic formulas may be used in two ways:

(1) Starting from c we may determine the corresponding N-interval and within that the relation between n and N.

(2) Starting from N we may determine the corresponding n and from n determine c.

The first method is useful for making a systematic tabulation of sampling plans whereas the second is suitable for computing "isolated" plans for a given N.

Starting from an integer value of c we first find n_c from (69) and the corresponding N_c from (61). Similarly we find $N_{c-0.5}$ and $N_{c+0.5}$, being the lower and upper limit for N having c as optimum acceptance number.

In the asymptotic solution we have disregarded the discreteness of c and n. We may, however, afterwards try to take the effect of the discreteness of c into account by investigating the relationship between n and N for given (integer) value of c. From $dR(N,n,c)/dn = 0$ it can be found that n is approximately a linear function of

$\log N$ with slope $-1/\log q_2$*. Within the interval $(N_{c-0.5}, N_{c+0.5})$ we may therefore determine n from the approximate formula

$$n = n_c - (\log N - \log N_c)/\log q_2, \quad N_{c-0.5} < N < N_{c+0.5}, \tag{70}$$

which for small p_2 and small intervals may be replaced by

$$n = n_c + (N - N_c)/N_c p_2, \quad N_{c-0.5} < N < N_{c+0.5}. \tag{71}$$

It follows that the values of n belong to the interval

$$n_c \pm \beta\left(\varphi_0 + \frac{1}{2n_c}\right)/2p_2.$$

For applications in practice we give the formula corresponding to (61) *with logarithms to base* 10, *i.e.*

$$\log(N_c - n_c) = \varphi n_c + \frac{1}{2}\log n_c + \delta \tag{72}$$

where

$$\varphi = p_0 \log\frac{p_0}{p_i} + q_0 \log\frac{q_0}{q_i}, \quad i = 1 \text{ or } 2, \tag{73}$$

$$\delta = -\log(\lambda\varphi_o), \tag{74}$$

and

$$\lambda\varphi_0 = 10^{\varphi\left(\alpha + \frac{\beta}{2}\right)} \frac{\varphi}{\log e}\sqrt{\frac{q_0}{2\pi p_0}} \sum_{i=1}^{2} \frac{\gamma_i p_i}{|p_0 - p_i|}\left(\frac{q_i}{q_0}\right)^{\alpha + \frac{\beta}{2}} \tag{75}$$

– *a having been replaced by* $\frac{\alpha}{\beta} + \frac{1}{2}$ *in* $\lambda\varphi_o$.

In the following we shall make much use of (72) with $N_c - n_c$ replaced by N_c which only means that we disregard terms of order n_c/N_c and less.

The approximation obtained by using (69), (70), and (72) is usually very good even for quite small values of c. *Normally the approximate value of c will deviate at most 1 from the correct value.* The approximation depends essentially on p_2/p_1, being good for large values of p_2/p_1 and poorer for small values. Two examples for $p_2/p_1 = 6.7$ and 3.3, respectively, will show the results obtained for a typical good and poor case. Table 1 and Fig. 3 show that the approximate and the exact solution are practically identical in the first case whereas the approximate solution in the second case often will lead to a value of c being 1 too large and a corresponding value of n.

* This results is due to Mrs. K. West Andersen.

TABLE 1.
Comparisons of exact and approximate sampling plans computed from (69), (70), and (72).

$p_r = p_s = 0.010,\ p_1 = 0.006,\ p_2 = 0.040,\ w_2 = 0.05.$
$\alpha = -26.7,\ \beta = 55.509,\ \varphi = 0.0034156,\ \delta = 1.5499,\ -1/\log q_2 = 56.405.$

c	Approximation			Exact	
	n_c	n	$N_c \pm 0.5$	n	N
1	57	43– 66	269– 714	45– 65	280– 714
2	112	104–120	715– 1400	105–120	715– 1420
4	223	216–230	2490– 4300	220–230	2550– 4390
6	334	328–340	7190– 12000	330–340	7390– 12300
8	445	439–451	19700– 32300	440–450	20200– 33000
10	556	550–562	52400– 85300	550–560	53600– 87000
12	667	661–673	137000–200000	665–670	140000–200000

$p_r = p_s = 0.010,\ p_1 = 0.006,\ p_2 = 0.020,\ w_2 = 0.05.$
$\alpha = -143.0,\ \beta = 85.879,\ \varphi = 0.0009088,\ \delta = 2.0785,\ -1/\log q_2 = 113.97.$

c	Approximation			Exact	
	n_c	n	$N_c \pm 0.5$	n	N
2	72	44– 88	715– 1750	–	–
4	243	234– 251	2790– 3970	245– 250	4420– 5590
6	415	408– 422	5410– 7150	415– 420	7100– 8980
8	587	581– 593	9270– 11900	585– 595	11300– 14200
10	759	753– 765	15100– 19000	755– 765	17700– 22000
12	931	925– 936	23800– 29600	930– 935	27300– 33700
14	1102	1097–1107	36800– 45700	1100–1105	41500– 51100
16	1274	1269–1279	56500– 69700	1270–1280	62800– 77200
18	1446	1441–1451	86000–106000	1445–1450	94600–116000
20	1618	1613–1623	130000–159000	1615–1620	142000–173000

It is *essential for the efficiency of the approximation to use the right relation between n and c*, see the discussion in section 12, and it is therefore fortunate that this relation is a simple linear one.

The approximation formula breaks down for values of N for which the cheapest solution is acceptance without inspection (or rejection without inspection). As will be seen from Table 1 the approximation formula may in such cases lead to a sampling plan even if no optimum plan exists. The difference in costs by using such a plan instead of accepting without sampling inspection will, however, normally be small.

Turning to the inverse formula (67) numerical investigations show that the results are not as accurate as those found from (61). Taking one more term in the inversion of (61) and changing to logarithms with base 10 we find

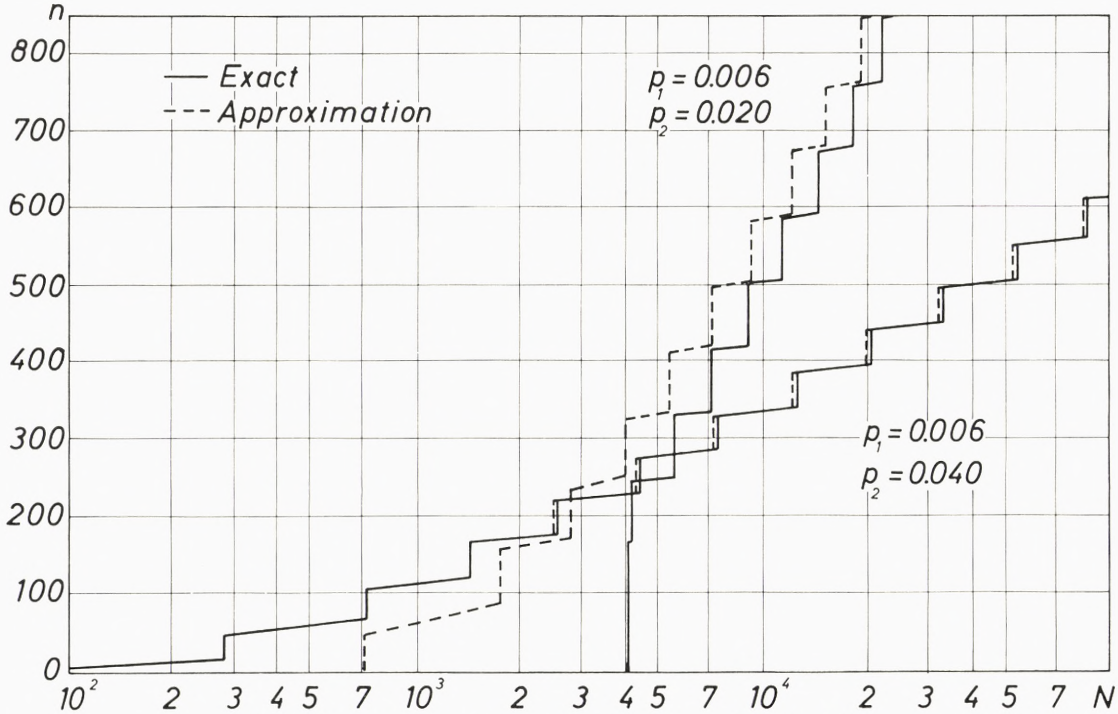

Fig. 3. Comparisons of exact and approximate sampling plans.

$$n_N = \frac{1}{\varphi}\left\{\log N - \left(\frac{1}{2}\log\log N + d\right)\left(1 - \frac{1}{3\log N}\right)\right\} \tag{76}$$

where

$$d = -\log\lambda\varphi_0 - \frac{1}{2}\log\varphi = \delta - \frac{1}{2}\log\varphi . \tag{77}$$

The exact inversion leads to the correction term $(\log e)/(2\log N) = 0.22/\log N$ which, however, on the basis of numerical investigations has been replaced by $1/(3\log N)$. If (76) is to be used extensively it pays to tabulate

$$g(N) = \log N - \frac{1}{2}\left(1 - \frac{1}{3\log N}\right)\log\log N \tag{78}$$

and use (76) in the form

$$n_N = \frac{1}{\varphi}\left\{g(N) - d\left(1 - \frac{1}{3\log N}\right)\right\}. \tag{79}$$

From n we may then find

$$c_N = p_0(n_N - \alpha) - \frac{1}{2}$$

TABLE 2.

Comparisons of exact and approximate sampling plans computed from (76).

$p_r = p_s = 0.010,\ w_2 = 0.05.$

N	$p_1 = 0.006,\ p_2 = 0.040.$ Approx. n_N	c_N	Exact n	c	$p_1 = 0.006,\ p_2 = 0.020.$ Approx. n_N	c_N	Exact. n	c
300	Accept		50	1				
500	20	0	60	1				
700	55	1	65	1				
1000	90	2	115	2				
2000	165	3	170	3				
3000	210	4	220	4	Accept		Accept	
5000	265	5	275	5	180	3	250	4
7000	300	5	285	5	320	5	335	5
10000	345	6	335	6	465	7	505	7
20000	420	8	395	7	755	10	760	10
30000	470	8	450	8	930	12	930	12
50000	525	9	505	9	1145	14	1105	14
70000	565	10	560	10	1290	16	1275	16
100000	610	11	610	11	1445	18	1445	18
200000	690	12	670	12	1750	22	1705	21

and round to the nearest integer. To obtain more accurate results n_c may be computed from the rounded value of c_N and n could then be found from (70) or (71).

Table 2 shows that (76) leads to good results for the two previously discussed typical examples.

As a general conclusion of the many numerical comparisons which have been carried out we may state that the asymptotic formulas give sufficiently good approximations to the optimum sampling plans for most practical purposes. If one wants to be sure to find the optimum plan one may start from the approximation and compare the costs of this plan with the costs of suitably chosen neighbouring plans thus finding the optimum one by trial and error.

The formulas (72) and (77) have, however, the serious drawback from the point of view of application that the constants δ and d are rather hard to compute. The asymptotic formulas have therefore in the following only been used to derive relationships between sampling plans under variation of the parameters. It is to be expected that these relationships will prove to be rather accurate in view of the good approximation demonstrated above.

According to (62) we have for the optimum plans that the average decision loss asymptotically is constant, i.e. $R-n \sim 1/\varphi_0$. For small N this gives an upper limit to the decision loss but the formula is not of much value as an approximation.

Fig. 4 sketches for the two previously considered examples $R-n$ as function of N. The discontinuities correspond to changes in c; each time c is increased by one n

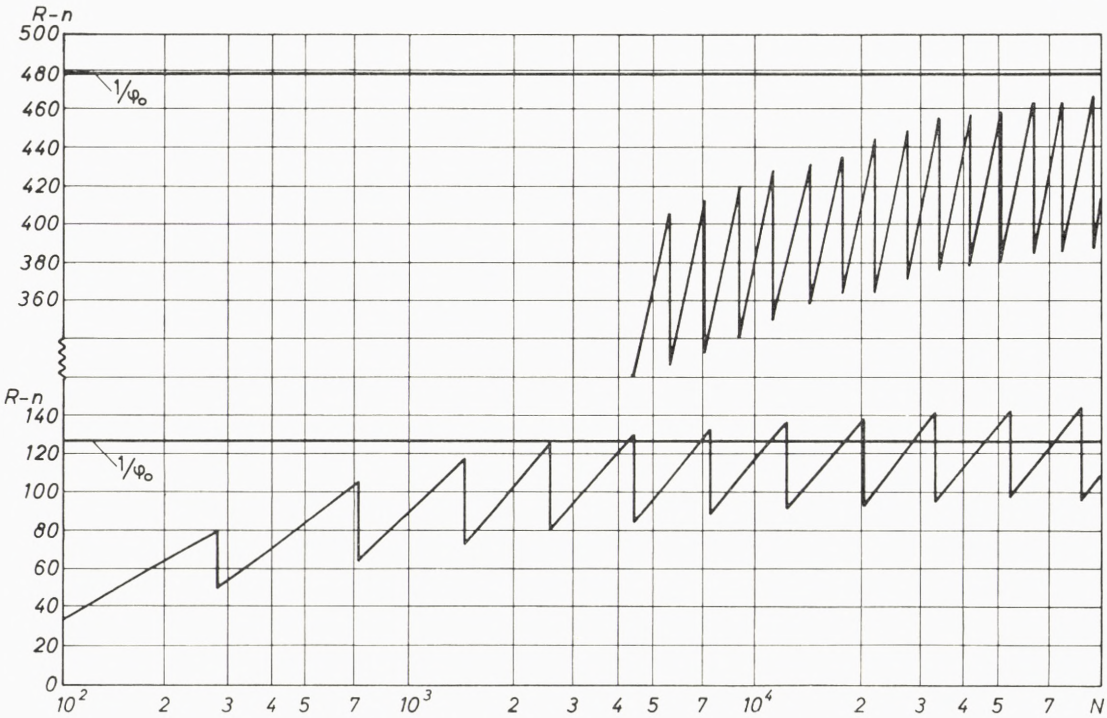

Fig. 4. Average decision loss as function of lot size.

increases approximately by β and $R-n$ decreases with the same quantity. The asymptotic result corresponds to the mid-points of the intervals. It will be seen that the asymptote is nearly being reached for $N = 100{,}000$ in the case with $p_2/p_1 = 6.7$ but not for $p_2/p_1 = 3.3$.

For small N a useful upper limit to the average decision loss may be obtained by noticing that $R < N\gamma_2$ if an optimum plan exists and the alternative is acceptance without inspection.

According to (63) and (64) the probabilities of wrong decisions, $Q(p_1)$ and $P(p_2)$, are asymptotically inversely proportional to N. Fig. 5 sketches for the two examples $Q(p_1)$ and $P(p_2)$ as functions of N. The asymptotic formula gives a reasonable approximation to $P(p_2)$ in both cases, whereas the approximation to $Q(p_1)$ is rather poor, particularly for the case $p_2/p_1 = 3.3$. The discontinuities resulting from changes of c are very pronounced for $Q(p_1)$.

6. Proportional change of (p_r, p_s, p_1, p_2) for fixed w_2

We shall first study the asymptotic formulas for all "quality levels" tending to zero with the same speed. Introducing the auxiliary quantities

$$\varrho_s = \frac{p_s}{p_r}, \quad \varrho_1 = \frac{p_1}{p_r}, \quad \varrho_2 = \frac{p_2}{p_r}, \quad \varrho_m = \frac{p_m}{p_r}, \quad \varrho = \frac{p_2}{p_1}, \tag{80}$$

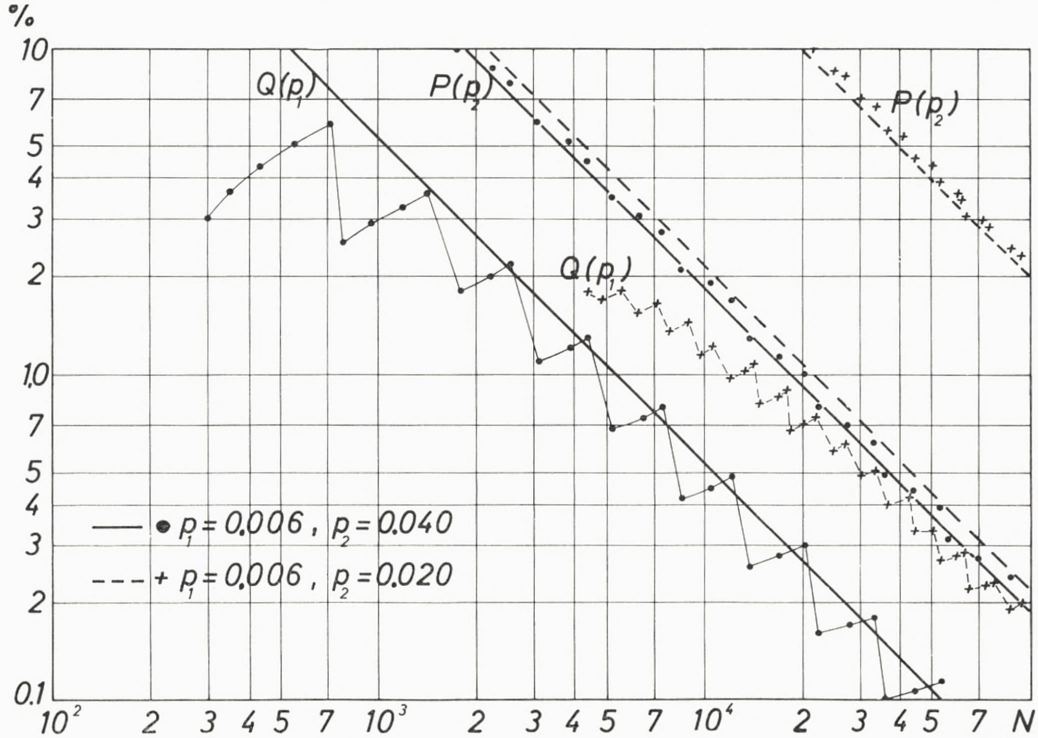

Fig. 5. Probabilities of wrong decisions as functions of lot size.

we find for $p_r \to 0$ and fixed $(\varrho_s, \varrho_1, \varrho_2, w_2)$

$$\alpha p_r \to \left(\ln \frac{w_2(\varrho_2 - 1)}{w_1(1 - \varrho_1)}\right) \Big/ (\varrho_2 - \varrho_1) = \alpha_0,$$

$$\beta p_r \to \left(\ln \frac{\varrho_2}{\varrho_1}\right) \Big/ (\varrho_2 - \varrho_1) = \beta_0,$$

$$p_0/p_r \to 1/\beta_0 = \varrho_0,$$

$$\varphi_0/p_r \to \varrho_0 \ln \frac{\varrho_0}{\varrho_i} + (\varrho_i - \varrho_0) = \varphi^*, \quad i = 1 \text{ or } 2,$$

and

$$\lambda \varphi_0/\sqrt{p_r} \to \exp\left\{\varphi^*\left(\alpha_0 + \frac{\beta_0}{2}\right)\right\} \frac{\varphi^*}{\sqrt{2\pi\varrho_0}} \sum_{i=1}^{2} \frac{w_i \varrho_i(\varrho_i - 1)}{(\varrho_s - \varrho_m)(\varrho_i - \varrho_0)} \exp\left\{(\varrho_0 - \varrho_i)\left(\alpha_0 + \frac{\beta_0}{2}\right)\right\} = \exp\{-\delta_0\},$$

where in the last expression $-a$ has been replaced by $\frac{\alpha}{\beta} + \frac{1}{2}$ as in (75).

Inserting these results into (69) and (72) we find

$$n_c p_r \rightarrow \alpha_0 + \beta_0\left(c + \frac{1}{2}\right) = n_0(c)$$

and

$$\ln(N_c p_r) \rightarrow \varphi^* n_0(c) + \frac{1}{2}\ln n_0(c) + \delta_0 = \ln N_0(c).$$

It follows that for small p_r we have approximately

$$n_c \sim n_0(c)/p_r$$

and

$$N_c \sim N_0(c)/p_r$$

where $n_0(c)$ and $N_0(c)$ are independent of p_r, i.e. *n and N vary inversely proportional to p_r for given c.*

Suppose that the optimum sampling plans have been tabulated for a small value of p_r, $p_r = 0.01$ say, and certain values of $(\varrho_s, \varrho_1, \varrho_2, w_2)$. The above result may then be used to find the optimum plans for λp_r, say, from the plans in the given table. Denoting the quantities required by $n_c(\lambda p_r)$ and $N_c(\lambda p_r)$ we have *for given c*

$$n_c(\lambda p_r) \sim n_c(p_r)/\lambda \tag{81}$$

and

$$N_c(\lambda p_r) \sim N_c(p_r)/\lambda, \tag{82}$$

i.e. we have found the following important "*proportionality law*":

The optimum sampling plan corresponding to $(N, \lambda p_r, \lambda p_s, \lambda p_1, \lambda p_2, w_2)$ is approximately equal to $(n^*/\lambda, c^*)$ where (n^*, c^*) is the plan corresponding to $(N^*, p_r, p_s, p_1, p_2, w_2)$ with $N^* = N\lambda$.

The theorem has been illustrated in Fig. 6 which shows that the approximation holds good also for quite large values of p_r.

This theorem greatly enlarges the field of application of the two master tables. The table with $p_r = 0.01$ may be used for $\lambda < 5$ and the table with $p_r = 0.10$ for $0.5 < \lambda < 2$, in that way covering all cases with $p_r < 0.20$ which is the domain of practical interest.

A large number of numerical investigations has shown that the proportionality law gives rather accurate results. The value of c found will seldom deviate more than 1 from the correct value. For $\lambda > 1$ the formula will normally tend to give too large a value of c and for $\lambda < 1$ too small a value.

The example in Table 3 shows the derivation of sampling plans with a break-even quality of $p_r = 0.03$, partly from the first master table using $\lambda = 3$ and partly from the second using $\lambda = 0.3$. Both results are remarkably close to the exact solution, see also Fig. 6.

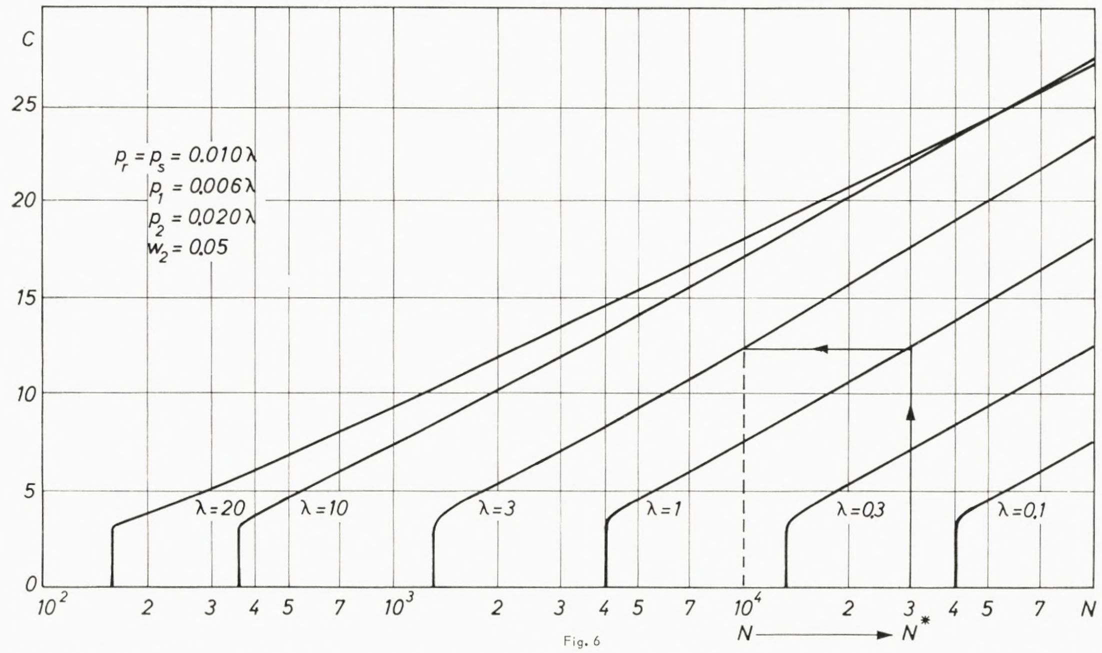

Fig. 6

Fig 6. Relation between lot size and acceptance number by proportional change of (p_r, p_s, p_1, p_2) for fixed w_2.

Table 3.

Comparisons of exact sampling plans for $p_r = p_s = 0.030$, $p_1 = 0.018$, $p_2 = 0.060$, $w_2 = 0.05$, and approximate plans derived from the master tables by the proportionality law.

N	Exact		Derived from $p_r = 0.01$ ($\lambda = 3$)			Derived from $p_r = 0.10$ ($\lambda = 0.3$)		
	n	c	$N^* = 3N$	$n^*/3$	c^*	$N^* = 0.3N$	$n^*/0.3$	c^*
1000	Accept		3000	Accept		300	Accept	
2000	110	5	6000	110	5	600	115	5
3000	140	6	9000	165	7	900	145	6
5000	225	9	15000	225	9	1500	200	8
7000	255	10	21000	255	10	2100	255	10
10000	310	12	30000	310	12	3000	285	11
20000	395	15	60000	395	15	6000	365	14
30000	455	17	90000	455	17	9000	425	16
50000	510	19	150000	540	20	15000	480	18
70000	570	21	210000	570	21	21000	535	20
100000	625	23	–	–	–	30000	565	21
200000	710	26	–	–	–	60000	675	25

Consider now the inverse formula (79). From

$$d + \log p_r \to \delta_0 \log e - \frac{1}{2} \log \varphi^* = d_0$$

we find

$$n_N(\lambda p_r) \sim \frac{n_N(p_r)}{\lambda} + \left(1 - \frac{1}{3 \log N}\right) \frac{\log \lambda}{\lambda \varphi(p_r)} \tag{83}$$

where $\varphi(p_r)$ denotes the value of φ for the given (basic) set of parameters. This formula shows how the sample size for a given lot size changes with the "quality level". This result is, however, not as accurate as the previous one for small N and it is neither as convenient for use in connection with the tables.

An example has been given in the following table for $N = 50{,}000$, $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, and $w_2 = 0.05$.

Comparisons of exact and approximate sampling plans derived from (83).

	Exact		Approximation	
λ	n	c	n	c
0.1	1850	3	2330	4
0.3	1300	7	1210	7
1.0	505	9	–	–
3.0	205	11	210	11
10.0	63	12	78	15

7. Change of p_s for fixed (p_r, p_1, p_2, w_2)

The master tables contain sampling plans for $p_s = p_r$ only, because a simple and rather accurate rule exists for deriving plans for $p_s \neq p_r$ from the tabulated ones.

From (69) and (72) it will be seen that p_s influences N_c only through δ. Writing

$$p_s - p_m = p_s - p_r + w_1(p_r - p_1) = w_1(p_r - p_1)\left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)$$

it follows from (72) that

$$\log N_c(p_r, p_s) = \log N_c(p_r, p_r) + \log\left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)$$

or

$$N_c(p_r, p_s) = N_c(p_r, p_r)/\lambda_s, \tag{84}$$

say, where

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)^{-1}. \tag{85}$$

TABLE 4.

Comparisons of exact sampling plans for $p_r = 0.010$, $p_s = 0.020$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$ with approximate plans derived from the master table.

N	Exact		Approximation		
	n	c	$N^* = 0.275N$	n^*	c^*
300	Accept		83	5	0
500	5	0	138	10	0
700	10	0	193	15	0
1000	15	0	275	15	0
2000	60	1	550	60	1
3000	110	2	825	110	2
5000	120	2	1380	120	2
7000	170	3	1930	170	3
10000	220	4	2750	220	4
20000	280	5	5500	280	5
30000	330	6	8250	330	6
50000	390	7	13800	390	7
70000	395	7	19300	395	7
100000	450	8	27500	445	8
200000	505	9	55000	550	10

We have thus proved the following *theorem*:

The optimum sampling plan corresponding to $(N, p_r, p_s, p_1, p_2, w_2)$ *is approximately equal to the plan* (n^*, c^*) *corresponding to* $(N^*, p_r, p_r, p_1, p_2, w_2)$ *with* $N^* = N\lambda_s$.

This theorem makes it possible to use the master tables also for $p_s \neq p_r$ if only N is replaced by N^*. The error in c by using this procedure will seldom be more than ± 1. An example has been given in Table 4 with

$$\lambda_s = \left(1 + \frac{2-1}{0.95(1-0.6)}\right)^{-1} = 0.275.$$

The corresponding "inverse" formula becomes

$$n_N(p_r, p_s) = n_N(p_r, p_r) + \frac{1}{\varphi}\left(1 - \frac{1}{3\log N}\right)\log\lambda_s. \tag{86}$$

Using this result for $N = 50{,}000$ and the parameters given in Table 4 we find

$$n = 505 - 293 \times 0.9291 \times 0.5607 = 350$$

as compared to the exact solution 390.

In the following sections we shall limit ourselves to consider cases with $p_s = p_r$ since we may always begin the analysis by replacing N by N^* if $p_s \neq p_r$. The "conversion factor" λ_s depends on w_2 and the ratios (ϱ_s, ϱ_1), i.e. λ_s is independent of p_2 and the general quality level.

8. Proportional change of (p_r, p_1, p_2) and change of w_2

Consider the problem of finding the optimum plans for an arbitrary set of parameter values (p_r, p_1, p_2, w_2) by combining the proportionality law with the relation between p_r and w_2 for given γ_2 and using the tabulated plans in the master table for parameter values $(p_{r0}, p_{10}, p_{20}, w_{20})$, say.

The problem is to determine λ so that $(p_r, p_1, p_2) = (\lambda p_{r0}^*, \lambda p_{10}, \lambda p_{20})$ and $(p_{r0}^*, p_{10}, p_{20}, w_2)$ give the same value of γ_2 as $(p_{r0}, p_{10}, p_{20}, w_{20})$. For this value of λ we may find the plans for (p_r, p_1, p_2, w_2) from the plans for $(p_{r0}^*, p_{10}, p_{20}, w_2)$ by means of the proportionality law, and the plans for $(p_{r0}^*, p_{10}, p_{20}, w_2)$ are identical to the plans for $(p_{r0}, p_{10}, p_{20}, w_{20})$. (It will be noted that p_{r0}^*/p_{r0} is identical to the function defined by (46)).

Since the value of γ_2 is the same for (p_r, p_1, p_2, w_2) and $(p_{r0}^*, p_{10}, p_{20}, w_2)$ we have the equation $\gamma_{20} = \gamma_2$ for the determination of λ, i.e.

$$\frac{w_{20}(p_{20} - p_{r0})}{w_{10}(p_{r0} - p_{10})} = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)}.$$

Introducing $p_{20} = p_2/\lambda$ and $p_{10} = p_1/\lambda$ we find

$$\lambda p_{r0} = \left(p_2 + \frac{w_{10}}{w_{20}}\gamma_2 p_1\right)\Bigg/\left(1 + \frac{w_{10}}{w_{20}}\gamma_2\right). \tag{87}$$

For the master table with $p_{r0} = 0.01$ and $w_{20} = 0.05$ the result is

$$\lambda = 100(p_2 + 19\gamma_2 p_1)/(1 + 19\gamma_2). \tag{88}$$

For the other master table ($p_{r0} = 0.10$) the factor 100 should be replaced by 10.

The results of sections 6–8 may be combined to the following *theorem*:

The optimum sampling plan corresponding to $(N, p_r, p_s, p_1, p_2, w_2)$ is approximately equal to $(n^*/\lambda, c^*)$ where (n^*, c^*) may be found in the master table for $N^* = N\lambda_s\lambda$, $p_{10} = p_1/\lambda$, and $p_{20} = p_2/\lambda$, the conversion factors being equal to

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)^{-1}$$

and

$$\lambda = 100(p_2 + 19\gamma_2 p_1)/(1 + 19\gamma_2), \quad \gamma_2 = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)},$$

for the 0.01-table, 100 being replaced by 10 for the 0.10-table.

By means of this theorem it is rather easy to find the optimum plan corresponding to an arbitrary set of parameter values if only p_1/λ and p_2/λ fall within the range of arguments in the master tables. If that is not the case the method given in the next section may be used.

Usually p_1/λ and p_2/λ will not be equal to the arguments used in the master tables. One might then interpolate but this is hardly worth while since the arguments in the table have been chosen in such a way that by rounding to the nearest argument the rounding error will ordinarily be less than 10 %.

If one wants to be sure to obtain a sufficiently large sample the value of p_1/λ should be rounded up and the value of p_2/λ rounded down.

As an example consider the problem of finding the sampling plans for $(p_r, p_1, p_2, w_2) = (0.03,\ 0.01,\ 0.07,\ 0.08)$ and $p_s = p_r$. Since $p_r < 0.05$, say, we choose to use the 0.01-table. From

$$\gamma_2 = \frac{8}{92}\,\frac{7-3}{3-1} = 0.174, \qquad 19\gamma_2 = 3.31,$$

we find $\lambda = (7+3.31)/(1+3.31) = 2.39$, $p_1/\lambda = 0.01/2.39 = 0.0042$, and $p_2/\lambda = 0.07/2.39 = 0.029$. The master table should thus be entered with $p_{10} = 0.004$ and $p_{20} = 0.030$. For $N = 2000$, say, we find $N^* = 4780$ and $(n^*, c^*) = (210,\ 3)$ leading to $(n, c) = (210/2.39,\ 3) = (90,3)$ which is the correct solution.

If $w_2 = 0.02$ instead of 0.08 we find similarly $\lambda = 4.38$, $p_1/\lambda = 0.0023 \simeq 0.0025$ and $p_2/\lambda = 0.0160 \simeq 0.0150$. For $N = 2000$ we get $N^* = 8760$ leading to acceptance without inspection as the most economical decision.

9. Change of w_2 for fixed (p_r, p_s, p_1, p_2)

In the following we shall develop a method for evaluating the effect of changing one of the five parameters only, and use it first for w_2 and then for p_r.

From (69) and (72) we find for given c

$$\frac{\partial n_c}{\partial \log w_2} = \frac{\partial \alpha}{\partial \log w_2} = 1\Bigg/\left(w_1 \log\frac{q_1}{q_2}\right) \tag{89}$$

and

$$\frac{\partial \log N_c}{\partial \log w_2} = \varphi \frac{\partial n_c}{\partial \log w_2} + \frac{1}{2}\frac{\partial \log n_c}{\partial \log w_2} + \frac{\partial \delta}{\partial \log w_2}. \tag{90}$$

The last term on the right hand side is a rather complicated function of the parameters. Tabulation of δ and graphical analysis of δ as a function of $\log w_2$ has shown, however, that a least for $w_2 \leqq 0.20$ and $p_r \leqq 0.10$ (and corresponding values of p_s, p_1, p_2) δ *is approximately a linear function of* $\log w_2$ *with a slope depending slightly on* $(\varrho_s, \varrho_1, \varrho_2)$ *and being practically independent of* p_r.

Limiting ourselves to the case $p_s = p_r$ we thus have

$$\frac{\partial \delta}{\partial \log w_2} \simeq -\, b_1(\varrho_1, \varrho_2),$$

say, where $b_1(\varrho_1, \varrho_2)$ has been tabulated in the appendix.

Writing $\delta = \delta(p_r, \varrho_1, \varrho_2, w_2)$ and putting $w_2 = 0.02$ and 0.20 respectively, so that $\Delta \log w_2 = \log 0.20 - \log 0.02 = 1$, an approximation to $\partial\delta/\partial \log w_2$ may be found as $\delta(p_r, \varrho_1, \varrho_2, 0.20) - \delta(p_r, \varrho_1, \varrho_2, 0.02)$. This approximation has been computed for both $p_r = 0.01$ and 0.10, and finally the average of the two has been taken as $-b_1$.

For small p_r we also have

$$\varphi/\log\frac{q_1}{q_2} \simeq \varphi^*/(\varrho_2 - \varrho_1) = b_2(\varrho).$$

The values given for b_2 have been computed as averages of $\varphi/\left(\log\frac{q_1}{q_2}\right)$ for $p_r = 0.01$ and $p_r = 0.10$.

For large n we have that $(\log e)/2n$ is small as compared to φ and we shall therefore disregard the second term on the right hand side of (90). We then have approximately

$$\frac{\partial \log N_c}{\partial \log w_2} = \frac{b_2(\varrho)}{w_1} - b_1(\varrho_1, \varrho_2)$$

which gives

$$N_c(w_2) = A w_2^{-b_1}\left(\frac{w_1}{w_2}\right)^{-b_2}$$

where A denotes a constant of integration. Changing from w_2 to λw_2 we get

$$N_c(\lambda w_2) = N_c(w_2)/f_1(\lambda) \tag{91}$$

where

$$f_1(\lambda) = \lambda^{b_1 - b_2}\left(1 - (\lambda - 1)\frac{w_2}{w_1}\right)^{b_2}. \tag{92}$$

From (69) we further have

$$n_c(\lambda w_2) = n_c(w_2) + g_1(\lambda) \tag{93}$$

where

$$g_1(\lambda) = \left(\log\frac{\lambda w_1}{1 - \lambda w_2}\right)\Big/\left(\log\frac{q_1}{q_2}\right) \sim \left(\ln\frac{\lambda w_1}{1 - \lambda w_2}\right)\Big/(\varrho_2 - \varrho_1)p_r. \tag{94}$$

For convenience f_1 and g_1 have been written as functions of λ only, even if they both depend also on other parameters. The function f_1 which will be called *the conversion factor for N due to a change in* w_2 has been tabulated in the appendix for $w_2 = 0.05$ as a function of $(\lambda, \varrho_1, \varrho_2)$. The function g_1 which gives *the correction to n due to a change in* w_2 has similarly been tabulated in the appendix as function of $(\lambda, \varrho_1, \varrho_2)$ for $w_2 = 0.05$ *and* $p_r = 0.01$. Values of this function for other values of p_r may be obtained as $g_1/100p_r$ where g_1 represents the tabulated values.

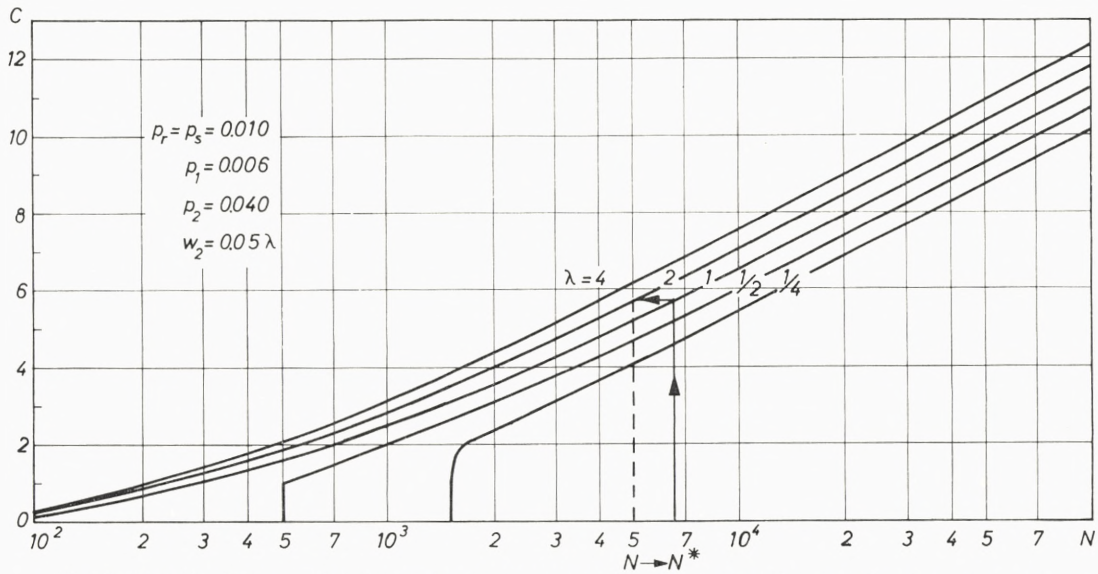

Fig. 7. Relation between lot size and acceptance number by change of w_2 for fixed (p_r, p_s, p_1, p_2).

The above results may be formulated as the following *theorem*:

The optimum sampling plan corresponding to $(N, p_r, p_s, p_1, p_2, \lambda w_2)$, $p_r = p_s$, *is approximately equal to* $(n^* + g_1(\lambda), c^*)$ *where* (n^*, c^*) *is the plan corresponding to* $(N^*, p_r, p_s, p_1, p_2, w_2)$ *with* $N^* = Nf_1(\lambda)$.

The theorem has been illustrated in Fig. 7.

This theorem enlarges the field of application of the two master tables with respect to values of w_2 in a similar manner as the law of proportionality does with respect to the other parameters. The results of using the approximation have been compared with the exact solutions in a large number of cases and the deviations found between the approximate and the correct value of c have never exceeded 1 for $\lambda < 4$. There is a tendency for the approximation to give too small a value of c for $\lambda > 1$ and too large a value for $\lambda < 1$, in particular for small N.

It should be noted that the formula breaks down in some cases for small N. Let N_a denote the largest N for which acceptance without inspection is cheaper than sampling inspection for the master table used. If $\lambda > 1$ and $Nf_1(\lambda) = N^* < N_a$ then the formula does not lead to a sampling plan even if there may exist a plan which for λw_2 is cheaper than acceptance without inspection. Similarly, for $\lambda < 1$ and $Nf_1(\lambda) = N^* > N_a$ there may be some cases where the approximation formula leads to a sampling plan even if the cheapest solution is acceptance without inspection.

An example has been shown in Table 5. The approximation is remarkably good. Since $N_a = 74$ the approximation formula leads to acceptance without inspection for all $N \leqq 57$. Sampling plans cheaper than acceptance without inspection do, however, exist for $12 \leqq N \leqq 57$.

Table 5.

Comparisons of exact sampling plans for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.10$, and approximate plans derived from the master table. $f_1 = 1.29$, $g_1 = 20$.

N	Exact		Approximate		
	n	c	$N^* = 1.29N$	$n^* + 20$	c^*
50	15	0	65	Accept	
70	20	0	90	25	0
100	25	0	129	30	0
200	35	0	258	35	0
300	75	1	387	75	1
500	85	1	645	85	1
700	130	2	903	130	2
1000	140	2	1290	140	2
2000	240	4	2580	240	4
3000	250	4	3870	250	4
5000	305	5	6450	300	5
7000	355	6	9030	355	6
10000	405	7	12900	405	7
20000	465	8	25800	465	8
30000	520	9	38700	520	9
50000	575	10	64500	575	10
70000	630	11	90300	630	11
100000	635	11	129000	635	11
200000	740	13	–	–	–

Using the method of section 8 we find $\gamma_2 = 0.833$, $\lambda = 0.804$, $p_1/\lambda = 0.0075$, and $p_2/\lambda = 0.050$. Since p_1/λ falls outside the range of arguments in the master table the method does not apply. Using $p_1/\lambda = 0.007$ gives, however, a rather good approximation.

From the inverse formula (79) we get

$$\frac{\partial n_N}{\partial \log w_2} = \frac{b_1(\varrho_1, \varrho_2)}{\varphi}\left(1 - \frac{1}{3 \log N}\right)$$

and consequently

$$n_N(w_2) = A + \frac{b_1}{\varphi}\left(1 - \frac{1}{3 \log N}\right)\log w_2$$

or

$$n_N(\lambda w_2) = n_N(w_2) + \frac{b_1}{\varphi}\left(1 - \frac{1}{3 \log N}\right)\log \lambda. \tag{95}$$

This shows that the difference between $n_N(\lambda w_2)$ and $n_N(w_2)$ for given N is proportional to $\log \lambda$. This formula is, however, not as accurate as (93) for small N.

An example has been given in the following table for $N = 20{,}000$, $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.020$, and $w_2 = 0.05$, which gives $b_1 = 0.59$ and $1/\varphi = 1100$.

Comparisons of exact and approximate sampling plans derived from (95).

		Exact		Approx.	
$100w_2$	λ	n	c	n	c
2.5	0.5	540	8	580	9
5.0	1.0	760	10	–	–
10.0	2.0	980	12	940	11
20.0	4.0	1130	13	1120	13

10. Change of $p_r = p_s$ for fixed (p_1, p_2, w_2)

From (69) and (72) we find for given c

$$\frac{\partial n_c}{\partial \log p_r} = \frac{\partial \alpha}{\partial \log p_r} = -p_r\left(\frac{1}{p_r - p_1} + \frac{1}{p_2 - p_r}\right)\bigg/\left(\log\frac{q_1}{q_2}\right) \tag{96}$$

and

$$\frac{\partial \log N_c}{\partial \log p_r} = \varphi\frac{\partial n_c}{\partial \log p_r} + \frac{1}{2}\frac{\partial \log n_c}{\partial \log p_r} + \frac{\partial \delta}{\partial \log p_r}. \tag{97}$$

Numerical investigations show that—for $w_2 < 0.20$, $p_r < 0.20$, and $p_r = p_s$ — *δ is approximately a linear function of $\log p_r$ with a slope depending on ϱ and being practically independent of "the level of (p_1, p_2)" and of w_2 if only p_r does not come too close to p_1 or p_2*, i.e.

$$\frac{\partial \delta}{\partial \log p_r} \simeq b_3(\varrho) \quad \text{for} \quad p_1\varrho^{1/5} < p_r < p_2\varrho^{-1/5},$$

say, where $b_3(\varrho)$ has been tabulated. (Another limitation of no practical importance is that p_2 must not be too close to 1). An approximation to $\partial\delta/\partial\log p_r$ may be computed as the corresponding difference-quotient setting $p_r = p_1\varrho^{1/5}$ and $p_r = p_2\varrho^{-1/5}$ respectively. This has been done for $w_2 = 0.05$ and for the "standard" values of p_1 and p_2, partly at the 1 % and partly at the 10 % level. The value of b_3 given in the table is the average of the two values found.

Proceeding as in section 9 we have approximately

$$\frac{\partial \log N_c}{\partial \log p_r} = -b_2(\varrho)p_r\left(\frac{1}{p_r - p_1} + \frac{1}{p_2 - p_r}\right) + b_3(\varrho)$$

which on integration gives

Table 6.

Comparisons of exact sampling plans for $p_r = p_s = 0.020$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$ with approximate plans derived from the master table. $\lambda = 2$, $f_2 = 0.52$, $g_2 = -55$.

N	Exact		Approximate		
	n	c	$N^* = 0.52N$	$n^* - 55$	c^*
2000	Accept		1040	60	2
3000	75	2	1560	110	3
5000	130	3	2600	165	4
7000	180	4	3640	170	4
10000	230	5	5200	225	5
20000	290	6	10400	280	6
30000	345	7	15600	335	7
50000	400	8	26000	390	8
70000	450	9	36400	445	9
100000	460	9	52000	450	9
200000	565	11	104000	555	11

$$N_c(p_r) = Ap_r^{b_3}\left(\frac{p_2 - p_r}{p_r - p_1}\right)^{b_2}$$

and

$$N_c(\lambda p_r) = N_c(p_r)/f_2(\lambda) \tag{98}$$

where

$$f_2(\lambda) = \lambda^{-b_3}\left(\frac{(\varrho_2 - 1)(\lambda - \varrho_1)}{(1 - \varrho_1)(\varrho_2 - \lambda)}\right)^{b_2}. \tag{99}$$

From (69) we further have

$$n_c(\lambda p_r) = n_c(p_r) + g_2(\lambda) \tag{100}$$

where

$$g_2(\lambda) \simeq \left(\ln\frac{(1 - \varrho_1)(\varrho_2 - \lambda)}{(\varrho_2 - 1)(\lambda - \varrho_1)}\right)\Big/(\varrho_2 - \varrho_1)p_r. \tag{101}$$

The conversion factor for N due to a change in $p_r, f_2(\lambda)$, has been tabulated in the appendix as function of $(\lambda, \varrho_1, \varrho_2)$, and *the correction to n due to a change in $p_r, g_2(\lambda)$*, has been tabulated as function of $(\lambda, \varrho_1, \varrho_2)$ for $p_r = 0.01$. Values of $g_2(\lambda)$ for other values of p_r may be found from the tabulated ones by dividing by $100p_r$.

The above results may be formulated as the following *theorem*:

The optimum sampling plan corresponding to $(N, \lambda p_r, \lambda p_s, p_1, p_2, w_2)$, $p_r = p_s$, is approximately equal to $(n^* + g_2(\lambda), c^*)$ where *(n^*, c^*) is the plan corresponding to $(N^*, p_r, p_s, p_1, p_2, w_2)$ with $N^* = Nf_2(\lambda)$.*

With the given set of tables this theorem is, however, not as important in practice as the previous ones, because the tables contain the optimum plans for so many combinations of (p_r, p_1, p_2) that an adjustment of the relative position of p_r within the interval (p_1, p_2) will seldom be felt necessary from a practical point of view.

In table 6 an example has been shown of the effect of changing $p_r = p_s$ from 0.010 to 0.020 within the interval $(p_1, p_2) = (0.006,\ 0.040)$.

From the inverse formula (79) we get

$$\frac{\partial n_N}{\partial \log p_r} = -\frac{b_3(\varrho)}{\varphi}\left(1 - \frac{1}{3\log N}\right)$$

which leads to

$$n_N(\lambda p_r) = n_N(p_r) - \frac{b_3}{\varphi}\left(1 - \frac{1}{3\log N}\right)\log\lambda. \tag{102}$$

An example of the application of this formula has been given in the following table for $N = 50{,}000$, $p_r = p_s = 0.010$, $\lambda = 0.5$ and 2.5, $p_1 = 0.002$, $p_2 = 0.040$, and $w_2 = 0.05$, which give $b_3 = 1.09$ and $1/\varphi = 177$.

Comparisons of exact and approximate sampling plans derived from (102).

		Exact		Approximate	
p_r	λ	n	c	n	c
0.005	0.5	350	4	370	4
0.010	1.0	315	4	–	–
0.025	2.5	250	4	245	4

11. Change of all parameters

The results of the preceding sections may be combined into a "chain formula" of the type

$$N_c(\lambda p_r, \varrho_s \lambda p_r, \lambda p_1, \lambda p_2, \lambda_1 w_2) = N_c(p_r, p_r, p_1, p_2, w_2)/\lambda_s f_1 \lambda \tag{103}$$

and

$$n_c(\lambda p_r, \varrho_s \lambda p_r, \lambda p_1, \lambda p_2, \lambda_1 w_2) = (n_c(p_r, p_r, p_1, p_2, w_2) + g_1)/\lambda \tag{104}$$

where

$$\lambda_s = \left(1 + \frac{\varrho_s - 1}{(1 - \lambda_1 w_2)(1 - \varrho_1)}\right)^{-1},$$

$f_1(\lambda_1)$ and $g_1(\lambda_1)$ being defined by (92) and (94) for $\varrho_1 = p_1/p_r$ and $\varrho_2 = p_2/p_r$.

In the master tables $p_r = p_s = 0.01$ (or 0.10) and $w_2 = 0.05$ have been used as reference values. What has been denoted by λ and λ_1 in the above formulas become $100\,p_r$ (or $10\,p_r$) and $20\,w_2$ if p_r and w_2 denote the values for which the optimum plan is required.

TABLE 7.

Comparisons of exact sampling plans for $p_r = 0.030$, $p_s = 0.060$, $p_1 = 0.018$, $p_2 = 0.120$, $w_2 = 0.10$ and approximate plans derived from the master table for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

N	Exact		Approximate		
	n	c	$N^* = 1.02N$	$(n^* + 20)/3$	c^*
50	Accept		51	Accept	
70	5	0	71	Accept	
100	5	0	102	10	0
200	10	0	204	10	0
300	10	0	306	25	1
500	25	1	510	25	1
700	30	1	714	30	1
1000	45	2	1020	45	2
2000	65	3	2040	65	3
3000	80	4	3060	80	4
5000	100	5	5100	100	5
7000	100	5	7140	100	5
10000	120	6	10200	120	6
20000	135	7	20400	155	8
30000	155	8	30600	155	8
50000	175	9	51000	175	9
70000	190	10	71400	195	10
100000	210	11	102000	210	11
200000	225	12	204000	230	12

We thus get the following rule for using the master table with $p_r = 0.01$:

The optimum plan for $(N, p_r, p_s, p_1, p_2, w_2)$ *with* $p_r < 0.05$ *is approximately equal to* $((n^* + g_1)/100\,p_r, c^*)$ *where* (n^*, c^*) *may be found by entering the master table with*

$$N^* = N(100\,p_r)f_1(20\,w_2, \varrho_1, \varrho_2)\Bigg/\left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right), \tag{105}$$

$\varrho_1 = p_1/p_r$, $\varrho_2 = p_2/p_r$, *and* $g_1 = g_1(20\,w_2, \varrho_1, \varrho_2)$, *the arguments for* (p_1, p_2) *in the master table being* $(\varrho_1/100, \varrho_2/100)$.

For $0.05 < p_r < 0.20$ the master table with $p_r = 0.10$ should be used accordingly.

If $(\varrho_1/100, \varrho_2/100)$ are not to be found in the table then use the "nearest" argument or interpolate. One may also use the results in section 10 to change p_r in the master table so that the relations between (p_r, p_1, p_2) in the table become closer to the ones for which the sampling plan is required. From a practical point of view, however, the master tables combined with the rule above will normally suffice.

An example has been given in Table 7. The conversion factor for N is found as

$$3f_1(2{,}0.6{,}4.0)\Bigg/\left(1+\frac{30}{0.90\times 12}\right) = 3\times 1.29/3.78 = 1.02.$$

The agreement between the approximate and the exact solution is very good. Using instead the method of section 8 we get $\lambda_s = 1/3.78 = 0.265$, $\lambda = 2.40$, $p_1/\lambda = 0.0075$, and $p_2/\lambda = 0.050$, i.e. $N^* = 0.636N$ and $n = n^*/2.40$. Since the master table does not contain the argument 0.0075 we may as an approximation use 0.0070 which, however, will tend to give too small values of c.

The corresponding inverse formula takes the form

$$\left.\begin{aligned} n_N(p_r, p_s, p_1, p_2, w_2) &= \frac{1}{100\,p_r}\left[n_0 + \frac{1}{\varphi}\left\{\log(100\,p_r) + b_1 \log(20\,w_2)\right.\right. \\ &\quad \left.\left. - \log\left(1+\frac{p_s - p_r}{w_1(p_r - p_1)}\right)\right\}\left(1-\frac{1}{3\log N}\right)\right] \end{aligned}\right\} \quad (106)$$

where n_0 denotes the sample size to be found in the master table for $p_r = 0.01$, $\varrho_1 = p_1/p_r$, $\varrho_2 = p_2/p_r$, corresponding to the given lot size N.

As an example consider the determination of n for $N = 50{,}000$ and the parameters given in Table 7. The value of $1/\varphi$ is 293 for $p_1 = 0.006$ and $p_2 = 0.040$, and $\left(1-\frac{1}{3\log N}\right) = 0.929$, so that we find

$$n = \frac{1}{3}(505 + 293(\log 3 + 0.61\log 2 - \log 3.78)0.929)$$

$$= \frac{1}{3}(505+23) = 176$$

in agreement with the (rounded) exact solution, $n = 175$, given in Table 7.

12. Efficiency

In a previous paper [6] it has been proposed to define the efficiency of a sampling plan as

$$e(N,n,c) = R_0(N)/R(N,n,c) \quad (107)$$

where $R_0(N)$ denotes the costs of the optimum plan and $R(N,n,c)$ denotes the costs of the plan in question.

We shall first discuss the efficiency of a sampling plan on the assumption that the optimum relationship between n and c has been used so that the loss in efficiency is due to using a wrong relationship between N and n. Looking at Fig. 2 it will be seen that it does not matter much whether we use the value of c giving the absolute minimum of R or a neighbouring value of c provided n is chosen such that a (relative) minimum of R is obtained.

For a given set of parameters let (n_0, c_0) be optimum for N_0 and (n_1, c_1) be optimum for N_1. From (26) it follows that

$$R(N, n, c) = n + (N - n)h(n, c)$$

where

$$h(n, c) = \gamma_1 Q(p_1) + \gamma_2 P(p_2).$$

Using the plan (n_1, c_1) for lot size N_0 (instead of N_1) we find

$$\left.\begin{aligned} R(N_0, n_1, c_1) &= n_1 + (N_0 - n_1)h(n_1, c_1) \\ &= n_1 + (N_0 - n_1)(R_0(N_1) - n_1)/(N_1 - n_1). \end{aligned}\right\} \quad (108)$$

It is therefore rather simple by means of the function $R_0(N)$ to evaluate the efficiency of plans contained in the master table in case such plans are used for the wrong value of N. The resulting efficiency is

$$e(N_0, n_1, c_1) = \frac{R_0(N_0)}{n_1 + (N_0 - n_1)(R_0(N_1) - n_1)/(N_1 - n_1)}. \quad (109)$$

Since $R_0(N) \sim n + 1/\varphi_0$ we have asymptotically

$$e(N_0, n_1, c_1) \sim \left(n_0 + \frac{1}{\varphi_0}\right) \Bigg/ \left(n_1 + \frac{N_0 - n_1}{N_1 - n_1} \frac{1}{\varphi_0}\right). \quad (110)$$

Introducing $n_0 = (\ln N_0)/\varphi_0 + o(\ln N_0)$ and considering n_1 as an arbitrary function of N_0, $n_1 = g(N_0)/\varphi_0$ say, we find

$$e(N, n_1, c_1) \sim (\ln N)/\left(g(N) + Ne^{-g(N)}\right) \quad (111)$$

for $n_1 = o(N)$, which is the result given without proof in [6].

For $g(N) = \lambda \ln N$ we get $e \to 1/\lambda$ for $\lambda \geqq 1$ but $e \to 0$ for $0 < \lambda < 1$, i.e. if we use a semilogarithmic relationship between n and N differing from the correct one then it is *important to use too large a sample*. For $g(N) = N^{\lambda}$, $\lambda > 0$, we get $e \to 0$.

A more accurate expression than (111) may be found by using all three term of (61) which leads to

$$e(N_0, n_1, c_1) \sim \left(n_0 + \frac{1}{\varphi_0}\right) \Bigg/ \left(n_1 + \frac{1}{\varphi_0}\left(e^{\varphi_0(n_0 - n_1)} \sqrt{\frac{n_0}{n_1}} + \frac{n_0 - n_1}{N_1 - n_1}\right)\right).$$

TABLE 8.

Investigation of efficiency for sampling plans with an acceptance number deviating 1 from the optimum. (e^* = asymptotic efficiency).

$p_r = p_s = 0.010,\ p_1 = 0.006,\ p_2 = 0.040,\ w_2 = 0.05.$

N	n_0	c_0	R	n_1	c_1	$100e$	$100e^*$
145	10	0	53	60	1	72	94
447	60	1	126	10	0	84	94
				115	2	86	95
1010	115	2	200	60	1	90	95
				170	3	92	96
1900	170	3	265	115	2	93	96
				225	4	95	97
3350	225	4	326	170	3	95	96
				280	5	96	97
5700	280	5	386	225	4	96	97
				335	6	97	98
9530	335	6	444	280	5	96	97
				390	7	97	98
15800	390	7	502	335	6	97	97
				445	8	98	98
25800	445	8	559	390	7	97	98
				500	9	98	98
42100	500	9	616	445	8	97	98
				555	10	98	98
68300	555	10	672	500	9	98	98
				615	11	98	98

This formula is, however, not of direct value because it contains N_1 which is unknown in practice. A simple and practically useful approximation is the following

$$e(N_0, n_1, c_1) \sim \left(n_0 + \frac{1}{\varphi_0}\right) \Big/ \left(n_1 + \frac{1}{\varphi_0} e^{\varphi_0 (n_0 - n_1)}\right). \tag{112}$$

This formula will, however, give too large efficiencies for small values of n because the decision loss has been overestimated.

In connection with the various approximations developed in the preceding sections is has repeatedly been stated that the value of c found by using the approximations will normally not deviate more than 1 from the correct value (for ($N < 200{,}000$). It is therefore of importance to know the efficiency of a plan for which $|c_1 - c_0| = 1$.

If $|c_1 - c_0| = a$ (constant) then $|n_1 - n_0| = a\beta$ and $e \to 1$ for $N_0 \to \infty$. Expanding the denominator of (112) we find for small values of $\varphi_0 a\beta$

$$e(N_0, n_1, c_1) \sim \left(n_0 + \frac{1}{\varphi_0}\right) \Big/ \left(n_0 + \frac{1}{\varphi_0} + \frac{1}{2}\varphi_0 a^2 \beta^2\right) \tag{113}$$

which converges rather fast to 1 for $n_0 \to \infty$ and $a = 1$. By means of the results of section 6 it will be seen that this asymptotic efficiency (as a function of c_0) is independent of the "quality level".

An example has been given in Table 8. The costs for each optimum plan have been compared with the costs of using a neighbouring plan, i.e. $c_1 = c_0 \pm 1$. The efficiency has been compared with the asymptotic efficiency found from (112). It will be seen that the efficiency is larger than 0.90 for $c \geqq 2$ and that the asymptotic formula gives too high an efficiency for small N. (N has been chosen as the geometric mean of the smallest and largest N for each c). This conclusion is typical for the cases investigated.

The conversion formulas and tables show how sensitive the solution is to changes of the parameters. A change of w_2 from 0.05 to 0.10, say, means, that N has to be multiplied by a factor of about 1.3 and the corresponding n should be increased by about 30. (In most systems of sampling plans in practical use to-day the same plan is used for a rather large N-interval, the ratio between endpoints usually being 1.5 or larger). As an example consider the case with $p_r = p_s = 0.010$, $p_1 = 0.006$ and $p_2 = 0.040$ as shown in the following table.

Optimum sampling plans.

	$w_2 = 0.05$		$w_2 = 0.10$	
N	n	c	n	c
500	60	1	85	1
1000	115	2	140	2
5000	275	5	305	5
10000	335	6	405	7
50000	505	9	575	10
100000	610	11	635	11

For most lot sizes we find the same value of c and a difference in n of about 25, in other cases the difference in c is 1 and the difference in n correspondingly larger. It is immediately clear that using the plans corresponding to $w_2 = 0.05$ if the true value of w_2 is 0.10 does not lead to an essential loss in efficiency.

The conclusion is that even if the value of w_2 used deviates from the true value by a factor of 2 the method will nevertheless lead to a sampling plan of very high efficiency.

Similar conclusions may be drawn for the other parameters by studying the conversion formulas.

The main reason why changes of p_r and w_2 does not affect the optimum solution seriously is that p_0 and φ_0 are independent of p_r and w_2.

Table 9.

Efficiencies of plans obtained by using wrong values of p_1, p_2 or both.

$p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

$100p_1$	0.6			0.7			0.5			0.6			0.6			0.7			0.5		
$100p_2$	4.0			4.0			4.0			3.5			5.0			3.5			5.0		
N	n	c	R	n	c	$100e$	n	c	$100e$	n	c	$100e$	n	c	$100e$	n	c	$100e$	n	c	$100e$
200	15	0	71	20	0	98	10	0	99	5	0	96	20	0	98	15	0	100	20	0	98
500	60	1	135	65	1	100	55	1	100	55	1	100	65	1	100	65	1	100	60	1	100
1000	115	2	199	120	2	100	110	2	100	115	2	100	110	2	100	120	2	100	105	2	99
2000	170	3	270	215	4	97	170	3	100	175	3	100	155	3	98	225	4	96	155	3	98
5000	275	5	372	320	6	96	235	4	98	295	5	98	205	4	92	345	6	94	210	4	94
10000	335	6	450	380	7	98	295	5	96	360	6	97	295	6	90	455	8	91	260	5	89
20000	395	7	532	480	9	97	355	6	94	480	8	95	340	7	82	570	10	89	310	6	84
50000	505	9	637	585	11	97	470	8	96	600	10	94	395	8	73	690	12	89	365	7	74
100000	610	11	718	690	13	95	530	9	93	720	12	92	445	9	65	805	14	87	415	8	66
200000	670	12	799	745	14	97	590	10	89	785	13	92	530	11	58	915	16	86	465	9	58

Since the most important relation in the system is

$$c + \frac{1}{2} = p_0(n - \alpha)$$

it is of importance to know how p_0 depends on p_1 and p_2.

From

$$\frac{\partial \ln p_0}{\partial \ln p_1} = \frac{p_0 - p_1}{q_1 \ln \frac{q_1}{q_2}} > 0 \text{ and } \frac{\partial \ln p_0}{\partial \ln p_2} = \frac{p_2 - p_1}{q_2 \ln \frac{q_1}{q_2}} > 0$$

it follows that p_0 is an increasing function of as well p_1 as p_2. Furthermore we have approximately

$$\frac{\partial \ln p_0}{\partial \ln p_1} + \frac{\partial \ln p_0}{\partial \ln p_2} \sim 1.$$

Within the domain of variation tabulated the first term is on the average 0.35 and the second 0.65.

The coefficient $p_0\alpha$ varies rather slowly with (p_1, p_2).

It follows that p_0 is known with a relative error of about the same size as the relative errors of p_1 and p_2.

If the choice of p_1 and p_2 is doubtful then *p_1 should be chosen too large and p_2 too small* (by about half of the percentage error in p_1) because the two errors will tend to counterbalance one another and thus give the correct p_0. The reason for bringing the two parameters closer together in case of doubt lies also in the fact that φ_0 is a decreasing function of p_1 and an increasing function of p_2. Since $n \sim (\log N)/\varphi_0$ the proposed rule will lead to a larger sample size than the optimum one which normally gives a better efficiency than too small a sample.

Table 9 shows the efficiency of using a plan obtained by entering the master table by a wrong value of p_1, p_2 or both. It is assumed that the true values of (p_1, p_2) are (0.006,0.040) and optimum plans have been substituted by plans obtained by using neighbouring values of (p_1, p_2) in the tables, i.e. the relative error of p_1 is 17 $^0/_0$ and the relative error of p_2 is 12.5 $^0/_0$ downwards and 25 $^0/_0$ upwards. The table shows that the efficiency in all cases is larger than 90 $^0/_0$ for $N < 10{,}000$. For $N = 200{,}000$, however, the efficiency falls to 58 $^0/_0$ in the worst case, i.e. the case where p_2 is chosen 25 $^0/_0$ too large.

The results in the table support the statement above that in case of doubt it is important to use a large value of p_1 and a small value of p_2.

A remark on the definition of efficiency. For a lot containing X defectives acceptance without inspection is cheaper than rejection without inspection for $X \leqq [Np_r]$. Classifying all lots in this way the average costs become

$$K_{Nm} = \sum_{X=0}^{[Np_r]} (NA_1 + XA_2) f_N(X) + \sum_{X=[Np_r]+1}^{N} (NR_1 + XR_2) f_N(X).$$

It is easily seen that $K_{Nm}/N \to k_m$ for $N \to \infty$, see (15), and that $K_{Nm} < Nk_m = K_m$.

It would be more correct to define efficiency as the ratio of costs in excess of K_{Nm} instead of K_m as in (107). This modification will increase the efficiencies for small N slightly whereas the above results regarding asymptotic efficiency will be unchanged. In Table 8 the first 5 efficiencies would be 83,87,89,91, and 93, whereas the remaining are unchanged, and in Table 9 the only change would be to increase 5 of the values of $100e$ for $N = 200$ by 1.

13. An example

Consider now an example starting from the original cost functions. To show the various aspects of the method the example will be worked out in more detail than is necessary for routine applications.

Let the three cost functions be $k_s(p) = 23 + 35p$, $k_r(p) = 16 + 35p$, $k_a(p) = 720p$, the coefficients denoting costs per item in cents, say, i.e. the costs of sampling and testing is 23 cents per item in the sample and the costs of accepting a defective item is 720 cents etc., see section 2.

Let us further assume that lots are generated with probability $w_1 = 0.93$ from a binomially controlled process with $p_1 = 0.009$ and with probability $w_2 = 0.07$ from a process with $p_2 = 0.080$.

The costs may then be described as in the following table:

w	p	$k_s(p)$	$k_r(p)$	$k_a(p)$	$k_m(p)$	$\|k_r(p) - k_a(p)\|$
0.93	0.009	23.315	16.315	6.480	6.480	9.835
0.07	0.080	25.800	18.800	57.600	18.800	38.800
Average	0.014	23.489	16.489	10.058	7.342	11.863

From (12) we find

$$p_r = (16-0)/(720-35) = 0.0234,$$

from (28)

$$p_m = 0.93 \times 0.009 + 0.07 \times 0.0234 = 0.0100,$$

and from (22)

$$p_s = (23-0)/(720-35) = 0.0336.$$

To find the optimum plan for $N = 500$ from the master table with $p_r = p_s = 0.010$ we first have to find the conversion factor λ_s which corrects for the difference between p_s and p_r, i.e.

$$\lambda_s^{-1} = 1 + \frac{336\text{–}234}{0.93(234\text{–}90)} = 1.76.$$

To use the method of section 8 we find $\gamma_2 = 0.296$, $\lambda = 1.97$, $p_1/\lambda = 0.0046 \simeq 0.005$, $p_2/\lambda = 0.041 \simeq 0.040$, and $N^* = 1.97\,N/1.76 = 1.12\,N = 560$. From the master table we read $(n^*,c^*) = (60,1)$ which gives $n = 60/1.97 = 30$ as the optimum sample size.

To illustrate the method of section 11 we have to find the conversion factor f_1 and the correction g_1 corresponding to the change from $w_2 = 0.05$ to 0.07. Since $\varrho_1 = 90/234 = 0.385 \simeq 0.40$ and $\varrho_2 = 800/234 = 3.42 \simeq 3.50$ we have $f_1 = 1.13$ and $g_1 = 15$. We then enter the master table with

$$N^* = N \times 2.34 \times 1.13/1.76 = 1.50\,N = 750$$

and find $(n^*,c^*) = (60,1)$ which finally gives $(n,c) = (30,1)$ since $(60+15)/2.34 = 32$.

To find the corresponding value of R we first compute

$$\gamma_1 = 0.93(234-90)/(336-100) = 0.567$$

and

$$\gamma_2 = 0.07(800-234)/(336-100) = 0.168$$

which lead to

$$R = n + (N-n)\big(0.567\,Q(p_1) + 0.168\,P(p_2)\big).$$

From a table of the binomial distribution one finds for $(n,c) = (30,1)$ that $Q(p_1) = 0.02982$ and $P(p_2) = 0.29579$ and consequently

$$R = 30 + 470 \times 0.0666 = 61.3.$$

The costs of sampling inspection and the average decision losses per lot are thus of nearly the same size.

Returning to the original monetary unit we find

$$k - k_m = R(k_s - k_m)/500 = 1.98$$

and finally

$$k = 7.34 + 1.98 = 9.32.$$

We thus have the following conclusion:

The quality of submitted lots is such that on the average costs per item will be 7.34 cents if all lots are classified correctly, i.e. all lots from process No. 1 are accepted and all lots from process No. 2 are rejected. To decide whether to accept or reject we inspect a sample of 30 items at the average costs of 0.97 cents per item of the lot. The decision losses will be 1.01 cents per item of the lot on the average. The first part of the costs, 7.34, depends on the prior distribution and can only be reduced by producing (or buying) lots of better quality. The second part, 1.98, depends on the sampling plan used. Since we have here used the optimum plan any change in sample size or acceptance number will result in increased costs. The average costs of accepting all lots without inspection are 10.06 cents per item.

The two functions $k_0(p) = K_0(p)/N$ and $k(p) = K(p)/N$ have been shown in Fig. 1 for the example above.

14. General remarks

There exists already a great body of theories and tables for constructing single sampling attribute plans based on *two specified quality levels* (p_1, p_2) and some further requirements. To see how the present paper fits into this the most important systems have been listed below by stating the "further requirements" for each system:

(a). Specification of the producer's and the consumer's risks, see for instance Peach and Littauer [7] and Grubbs [8].

(b). Specification of the consumer's risk and minimization of the average amount of inspection for lots of process average quality (p_1) in the case of rectifying inspection, see Dodge and Romig [9].

(c). Specification of the consumer's risk and minimization of the average costs for lots of process average quality (p_1), i.e. a generalization of the Dodge-Romig LTPD system requiring specification of one cost parameter, see Hald [10].

(d). Specification of two cost parameters, p_r and p_s, and a weight, w_2, and minimization of the average costs, as for instance in the present paper.

It follows from the results of the present paper that *from an economic point of view it is not advisable to fix the consumer's or the producer's risk*. On the contrary *the producer's and the consumer's risks should both tend to zero with increasing lot size.* This theorem is valid not only for the double binomial prior distribution but for any prior distribution, and it is valid not only for the Bayes solution but also for the minimax solution $(p_1 < p_r < p_2)$, the only difference being the speed of the convergence. For a discrete prior distribution the risks tend to zero inversely proportional to N, see (63) and (64). These considerations lead to the result that if one wants a system with a fixed risk then *the risk should be fixed to 50 per cent at a point between p_1 and p_2*. We may therefore increase the list of systems of sampling plans above by the following item:

(e). Minimization of average costs for lots of process average quality (p_1) under the restriction that $P(p_0) = 1/2$. Such a system, named *the IQL system* (*I*ndifference *Q*uality *L*evel) has been discussed by Hald, see [6] and [10], and will be further discussed in a forthcoming paper. This system requires the specification of p_0 and a cost parameter. In view of the asymptotic relation (66) between c and n it is clear that p_0 should be determined from (52).

The simplest possible system based on *the specification of two risks and having the same properties as the Bayes solution* may be formulated as follows:

(f). Specification of the consumer's *or* the producer's risk as inversely proportional to lot size, and $P(p_0) = 1/2$.

This system requires only the specification of one parameter (besides the two quality levels) and it is extremely simple to handle both mathematically and numerically. This is due to the fact that the equation $P(p_0) = 1/2$ has the solution $c = np_0 + (p_0 - 2)/3$ (with sufficient accuracy for all practical applications, perhaps apart from the case $c = 0$ where the exact solution may be easily found) and that the other equation, $Q(p_1) = \alpha/N$ say, may be solved with respect to N for related values of (n,c) from the first equation. Setting $c = 0,1,2,\ldots$ and solving the first equation for n, the second equation gives $N = \alpha/(1 - B(c,n,p_1))$ which may easily be found by means of a table of the binomial (or the Poisson) distribution. The only difficulty lies in the choice of α. If the problem is fully specified one may naturally choose α as the coefficient of $1/N$ in (63) and the system will then asymptotically give an approximation to the Bayes solution. The reason for using the simple system will, however, usually be that some of the parameters in the problem are unknown and in that case the choice of α will to some extent be arbitrary, just as in the other cases the choice of the producer's or the consumer's risk is arbitrary. This system of sampling plans will be discussed in more detail in the forthcoming paper on the *IQL* system.

Turning to applications it is important to notice that a system of sampling plans in practice often is required to serve several purposes. In particular we shall here stress (a) that the system should protect the consumer against deterioration of the prior distribution, (b) that the system should work as an incentive for the producer to produce better quality or at least to keep to the quality agreed upon, see Hill [11], and (c) that (average) costs should be minimized. The first two requirements are concerned with consequences of changes of the prior distribution and the problem should therefore really be formulated as a dynamic one. However, since a dynamic model at present is lacking we shall try to indicate how the Bayesian solution may be modified to take requirements (a) and (b) into account.

One of the arguments advanced against the Bayesian method in general has been that a prior distribution does normally not exist. This may be true in many fields but certainly not for industrial mass production with its effective planning and control of operations. Admittedly the prior distribution may change, but changes are usually rather small and slow within a given production period in which the same machinery, techniques, and raw materials are being used. We are here not concerned about isolated very poor lots which may occasionally occur since any sampling plan will detect such lots.

Published data on prior distributions are scarce. Whether the double binomial distribution is a reasonable approximation to distributions occurring in practice is not known. According to the experience of the author mixed binomial distributions with beta-distributions as weight functions are rather common. (A paper analogous to the present one will present the corresponding theory and tables for the beta-distribution).

One of the drawbacks of the Bayesian solution from a practical point of view is that the solution may be acceptance (or rejection) without inspection. If one is not completely confident that the prior distribution used is the right one and is stable, then a sampling plan is required to guard against deterioration of the prior distribution. One possibility is to use the first or one of the first Bayesian sampling plans in the appropriate table. If that is not satisfactory one may in such cases use an *IQL* plan.

The same procedure may be used to satisfy requirement (a) above. It should first of all be noted that if a Bayesian *sampling* plan exists then *some* protection against deterioration of the prior distribution is automatically obtained and the protection may in the usual way be expressed by means of the *OC* curve. It is always easy when the plan has been found to compute the consumer's risk and then to decide whether the risk is sufficiently small. If the consumer's risk is too large one may again find a sufficiently large sample in the same table or turn to an *IQL* plan or a *LTPD* plan.

The price to be paid for obtaining the required protection is naturally that the plan used will not minimize costs if the prior distribution holds. If the change in the value of c is not large the increase in costs will, however, be small.

For large lots the consumer's risk for the Bayesian sampling plan will usually be much smaller than 10 per cent so that the problem does only exist for small lots.

The incentive for the producer to keep to the specified quality is usually obtained by *alternating between normal and tightened inscpection* in a specific way such that the system reacts upon observed changes in the prior distribution. If it was possible to estimate in what way the distribution had changed the reaction could be made to depend on the change. In practice, however, one want to install tightened inspection as soon as possible on the basis of some over-all criterion, for example when the number of lots rejected exceeds some critical limit. A thorough theory does not exist but some rules have been found to work satisfactory in practice. The Military Standard 105 D uses the same sample size for normal and tightened inspection and a reduced acceptance number, c_T, for tightened inspection. The difference between the two acceptance numbers, $c_N - c_T$, equals 1 for $2 \leqq c_N \leqq 4$, 2 for $5 \leqq c_N \leqq 20$, and 3 for $c_N \geqq 21$. For $c_N = 0$ or 1, c_T is usually equal to c_N but the sample size is increased for tightened inspection. Similar rules may be used for the present tables although it has to be realized that the resulting plans will not be minimum-cost plans. The main point is, that under normal conditions the plans will minimize costs and that the plans may be adjusted to changes in the prior distribution so that costs are minimized under the new conditions. If, however, the incentive aspect of sampling inspection is more important for the user of the system than to minimize costs in case of change of the distribution then some form of tightened inspection may be introduced with the result that during periods of tightened inspection the plans will not minimize costs.

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Appendix

Master Tables of Sampling Plans

Tables of Conversion Factors

Summary of Conversion Formulas

All sampling plans in the master tables assume $p_r = p_s$ and $w_2 = 0.05$. In the first set of tables $p_r = 10\,\%$ and (p_1, p_2) take on the values

$$p_1 = 2.0,\ 2.5,\ 3.0,\ 3.5,\ 4.0,\ 5.0,\ 6.0,\ 7.0\,\%$$
$$p_2 = 15.0,\ 17.5,\ 20.0,\ 25.0,\ 30.0\,\%.$$

In the second set of tables $p_r = 1\,\%$ and (p_1, p_2) take on the values

$$p_1 = 0.20,\ 0.25,\ 0.30,\ 0.35,\ 0.40,\ 0.50,\ 0.60,\ 0.70\,\%$$
$$p_2 = 1.50,\ 1.75,\ 2.00,\ 2.50,\ 3.00,\ 3.50,\ 4.00,\ 5.00,\ 6.00,\ 7.00\,\%.$$

Single Sampling Tables for $p_1 = 2.0\,\%$

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$			$p_2 = 30.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 1460	Accept		1– 665	Accept		1– 334	Accept		1– 140	Accept		1– 45	Accept	
1460– 1530	16	2	666– 701	16	2	335– 414	7	1	141– 149	6	1	46– 92	1	0
1600– 1840	29	3	829– 1020	18	2	479– 586	17	2	191– 259	8	1	93– 103	6	1
2160– 2630	31	3				727– 946	19	2				139– 199	8	1
			1170– 1270	29	3				358– 396	16	2			
2630– 2760	43	4	1510– 1850	31	3	1020– 1240	29	3	509– 682	18	2	316– 346	15	2
3180– 3760	45	4	2270– 2710	43	4	1540– 1980	31	3	945– 1200	27	3	471– 679	17	2
4560– 4770	58	5	3280– 4100	45	4	2200– 2530	41	4	1590– 2240	29	3	974– 1400	25	3
5530– 6550	60	5				3120– 3990	43	4				2030– 2800	27	3
			4410– 4810	56	5				2380– 2720	37	4			
7990– 9560	74	6	5760– 7080	58	5	4690– 5040	53	5	3560– 4880	39	4	2800– 3970	34	4
11300– 14000	76	6	8530– 10000	70	6	6200– 7850	55	5	5910– 7790	48	5	5750– 7850	36	4
14000– 16400	89	7	12200– 15200	72	6	9860– 12100	66	6	10500– 14200	50	5	7850– 11000	43	5
19500– 24300	91	7				15300– 20400	68	6				15900– 21600	45	5
			16400– 17400	83	7				14200– 16800	58	6			
24300– 28200	104	8	20900– 25700	85	7	20400– 23600	78	7	22200– 30800	60	6	21600– 29800	52	6
33300– 40300	106	8	31300– 35800	97	8	29400– 37800	80	7	34600– 47100	69	7	43000– 58900	54	6
42500– 48000	119	9	43400– 54100	99	8	42500– 45600	90	8	63800– 81900	71	7	58900– 80000	61	7
56700– 68400	121	9				56400– 71700	92	8				115000–159000	63	7
			59700– 73500	111	9				81900– 99600	79	8			
73800– 81600	134	10	90200–113000	113	9	87600–108000	103	9	132000–200000	81	8	159000–200000	70	8
96200–116000	136	10	113000–125000	124	10	136000–178000	105	9						
128000–138000	149	11	151000–188000	126	10	178000–200000	115	10						
163000–200000	151	11												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 2.5\,\%$

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$			$p_2 = 30.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 1600	Accept		1– 698	Accept		1– 367	Accept		1– 139	Accept		1– 51	Accept	
1600– 1870	26	3	699– 791	15	2	368– 417	6	1	140– 161	6	1	52– 84	1	0
2050– 2440	39	4	944– 1040	26	3	418– 485	15	2	221– 291	8	1	85– 105	6	1
2930– 3120	41	4	1250– 1610	28	3	605– 777	17	2				150– 246	8	1
									292– 347	15	2			
3120– 3280	52	5	1610– 1960	39	4	778– 876	26	3	459– 673	17	2	247– 276	14	2
3840– 4640	54	5	2410– 2780	41	4	1090– 1410	28	3	674– 867	25	3	382– 570	16	2
4860– 5150	66	6	2780– 3050	51	5	1490– 1890	38	4	1170– 1520	27	3	670– 856	23	3
6060– 7330	68	6	3700– 4790	53	5	2410– 2810	40	4				1230– 1690	25	3
									1520– 2070	35	4			
7620– 8070	80	7	4790– 5670	64	6	2810– 3220	49	5	2860– 3290	37	4	1690– 1840	31	4
9520– 11500	82	7	6980– 8250	66	6	4050– 5260	51	5	3290– 3660	44	5	2580– 4100	33	4
12000– 12600	94	8	8250– 8690	76	7	5260– 6750	61	6	4850– 7000	46	5	4100– 5330	40	5
14900– 18100	96	8	10500– 13200	78	7	8680– 9750	63	6				7730– 10200	42	5
									7000– 8350	54	6			
18700– 19700	108	9	14100– 15900	89	8	9750– 11200	72	7	11200– 15000	56	6	10200– 15400	49	6
23300– 28200	110	9	19500– 24000	91	8	14200– 18000	74	7	15000– 18900	64	7	23800– 30900	57	7
29400– 30600	122	10	24000– 29100	102	9	18000– 23200	84	8	25700– 31700	66	7	45000– 57800	59	7
36200– 43900	124	10	36100– 40800	104	9	29900– 33000	86	8						
									31700– 42500	74	8	57800– 87700	66	8
46000– 56200	137	11	40800– 43700	114	10	33000– 37900	95	9	58700– 66100	76	8	133000–173000	74	9
68100– 71800	139	11	53300– 68800	116	10	48000– 60400	97	9	66100– 71800	83	9			
71800– 87000	151	12	68800– 79300	127	11	60400– 77600	107	10	95300–137000	85	9			
105000–112000	153	12	97700–117000	129	11	100000–110000	109	10						
									137000–159000	93	10			
112000–134000	165	13	117000–144000	140	12	110000–126000	118	11						
163000–174000	167	13	179000–200000	142	12	159000–200000	120	11						
174000–200000	179	14												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 3.0\,\%$

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$			$p_2 = 30.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 1760	Accept		1– 749	Accept		1– 396	Accept		1– 138	Accept		1– 57	Accept	
1760– 2100	35	4	750– 823	14	2	397– 481	14	2	139– 178	6	1	58– 78	1	0
2420– 2960	48	5	824– 980	24	3	636– 730	24	3	248– 299	14	2	79– 109	6	1
3450– 4180	61	6				923– 1110	26	3	404– 523	16	2	167– 209	8	1
			1250– 1340	35	4									
4960– 5910	74	7	1630– 1990	37	4	1110– 1340	35	4	524– 602	23	3	210– 298	14	2
7160– 8350	87	8	1990– 2200	47	5	1730– 1920	37	4	807– 1060	25	3	454– 503	16	2
10300– 11800	100	9	2710– 3150	49	5	1920– 2440	46	5	1060– 1150	32	4	504– 708	22	3
14200– 15000	102	9							1530– 2110	34	4			
			3150– 3600	59	6	3250– 3470	56	6				1130– 1590	30	4
15000– 16500	113	10	4460– 4970	61	6	4370– 5550	58	6	2110– 2830	42	5	2520– 3460	38	5
20000– 21700	115	10	4970– 5840	71	7	5550– 6120	67	7	4080– 5140	51	6	5500– 7410	46	6
21700– 23200	126	11	7300– 7820	73	7	7760– 9380	69	7	7120– 7910	53	6	11100– 12000	48	6
27900– 31400	128	11												
			7820– 9440	83	8	9380– 10700	78	8	7910– 9280	60	7	12000– 15700	54	7
31400– 39000	140	12	12200– 15200	95	9	13700– 15800	80	8	12700– 15200	62	7	23500– 25700	56	7
45200– 54400	153	13	19100– 24400	107	10	15800– 18700	89	9	15200– 16600	69	8	25700– 33000	62	8
65000– 75800	166	14				24200– 26400	91	9	22400– 29000	71	8	49100– 54800	64	8
			29700– 31700	118	11									
93300–106000	179	15	39100– 46300	120	11	26400– 32600	100	10	29000– 39500	79	9	54800– 68800	70	9
128000–134000	181	15	46300– 50400	130	12	44000– 56500	111	11	54600– 69500	88	10	102000–116000	72	9
134000–147000	192	16	62500– 71800	132	12	73100– 77200	121	12	97000–103000	90	10	116000–143000	78	10
178000–193000	194	16				97800–121000	123	12						
			71800– 80000	142	13				103000–122000	97	11			
193000–200000	205	17	99600–111000	144	13	121000–133000	132	13	168000–194000	99	11			
			111000–127000	154	14	169000–200000	134	13	194000–200000	106	12			
			159000–172000	156	14									
			172000–200000	166	15									

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 3.5\%$

$p_2 = 15.0\%$			$p_2 = 17.5\%$			$p_2 = 20.0\%$			$p_2 = 25.0\%$			$p_2 = 30.0\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 1850	Accept		1– 787	Accept		1– 399	Accept		1– 138	Accept		1– 62	Accept	
1850– 2030	32	4	788– 906	22	3	400– 470	13	2	139– 215	6	1	63– 71	1	0
2030– 2150	43	5	1060– 1290	33	4	549– 591	22	3	216– 252	13	2	72– 80	5	1
2580– 2690	45	5	1550– 1840	44	5	746– 886	24	3	342– 426	15	2	115– 175	7	1
2690– 3210	56	6	2270– 2610	55	6	887– 1150	33	4	427– 531	22	3	176– 227	13	2
3620– 4040	68	7	3340– 3670	66	7	1420– 1750	43	5	741– 804	24	3	340– 398	15	2
4910– 6120	81	8	4580– 4960	68	7	2250– 2620	53	6	805– 1070	31	4	399– 569	21	3
6660– 7660	93	9	4960– 6360	78	8	3400– 3570	55	6	1480– 2110	40	5			
												833– 940	28	4
9030– 9610	105	10	7260– 8830	89	9	3570– 3890	63	7	2660– 2990	48	6	1370– 1710	30	4
11600– 12300	107	10	10600– 12200	100	10	4990– 5660	65	7	4110– 4760	50	6	1710– 2140	36	5
12300– 14500	118	11	15400– 16900	111	11	5660– 7300	74	8	4760– 5680	57	7	3230– 3450	38	5
16700– 18100	130	12	21200– 22600	113	11	8840– 10700	84	9	7930– 8430	59	7			
												3450– 4850	44	6
22600– 27200	143	13	22600– 29000	123	12	13800– 15500	94	10	8430– 10700	66	8	6950– 11000	52	7
30700– 33800	155	14	32800– 39700	134	13	20100– 21600	96	10	14900– 20200	75	9	13600– 16600	59	8
41500– 50800	168	15	47600– 54400	145	14	21600– 28900	105	11	26400– 38000	84	10	24900– 26600	61	8
56200– 62900	180	16				33400– 41600	115	12						
			68700– 74600	156	15				45700– 51700	92	11	26600– 36500	67	9
76200– 94600	193	17	93600–100000	158	15	51600– 59900	125	13	71400– 79600	94	11	52400– 80700	75	10
103000–117000	205	18	100000–127000	168	16	79400– 86400	135	14	79600– 95600	101	12	101000–120000	82	11
139000–175000	218	19	144000–173000	179	17	112000–123000	137	14	138000–177000	110	13	180000–196000	84	11
187000–200000	230	20												
						123000–159000	146	15				196000–200000	90	12
						190000–200000	156	16						

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 4.0\,\%$

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$			$p_2 = 30.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 1950	Accept		1– 806	Accept		1– 402	Accept		1– 134	Accept		1– 65	Accept	
1950– 2130	40	5	807– 923	21	3	403– 447	12	2	135– 151	5	1	66– 79	5	1
2260– 2850	52	6	924– 981	30	4	487– 576	21	3	198– 276	13	2	119– 156	7	1
2850– 3090	63	7	1260– 1470	41	5	736– 944	31	4	359– 456	21	3	157– 244	13	2
3670– 4130	75	8	1770– 2210	52	6	1110– 1190	40	5	632– 719	29	4	326– 442	20	3
4730– 5520	87	9	2470– 2650	62	7	1530– 1670	42	5	996– 1100	31	4	640– 760	27	4
6120– 7390	99	10	3420– 3930	73	8	1670– 1880	50	6	1100– 1490	38	5	1140– 1240	29	4
7920– 10200	111	11	4780– 5830	84	9	2490– 2950	60	7	1860– 2220	46	6	1240– 1860	35	5
10200– 10800	122	12	6660– 8660	95	10	3730– 4610	70	8	3130– 4530	55	7	2320– 3010	42	6
13200– 14400	134	13	9170– 10100	105	11	5550– 7190	80	9	5210– 6550	63	8	4300– 4870	49	7
17000– 19100	146	14	12700– 14900	116	12	8210– 11200	90	10	8620– 9530	71	9	7280– 7980	51	7
22000– 25500	158	15	17500– 21900	127	13	12000– 13200	99	11	13300– 14200	73	9	7980– 11400	57	8
28400– 33900	170	16	24100– 25500	137	14	17500– 20300	109	12	14200– 18900	80	10	14600– 18000	64	9
36600– 45100	182	17	33000– 37200	148	15	25700– 31100	119	13	23400– 27100	88	11	26800– 42500	72	10
47200– 60400	194	18	45500– 54300	159	16	37700– 47600	129	14	38300– 53700	97	12	48500– 65700	79	11
60400– 64700	205	19	62600– 79400	170	17	55200– 73000	139	15	62500– 76100	105	13	87600–103000	86	12
77800– 85600	217	20	85500– 91700	180	18	79900– 85600	148	16	103000–151000	114	14	158000–200000	94	13
100000–113000	229	21	117000–133000	191	19	115000–130000	158	17	165000–200000	122	15			
129000–150000	241	22	160000–200000	202	20	168000–200000	168	18						
165000–200000	253	23												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 5.0\,\%$

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$			$p_2 = 30.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 2090	Accept		1– 822	Accept		1– 404	Accept		1– 123	Accept		1– 56	Accept	
2090– 2110	45	6	823– 949	27	4	405– 506	19	3	124– 160	5	1	57– 74	5	1
2110– 2240	55	7				553– 716	28	4	161– 230	12	2	121– 176	12	2
2480– 2750	66	8	950– 1001	36	5				268– 304	19	3	236– 359	19	3
2950– 3400	77	9	1210– 1340	46	6	766– 998	37	5	432– 526	27	4	424– 713	26	4
			1560– 1790	56	7	1060– 1370	46	6						
3520– 4210	88	10	2000– 2390	66	8	1450– 1860	55	7	685– 888	35	5	714– 825	32	5
4210– 5050	99	11	2570– 3280	76	9	1980– 2510	64	8	1060– 1480	43	6	1220– 1470	39	6
5050– 5230	109	12				2690– 3360	73	9	1630– 2420	51	7	2050– 2600	46	7
6060– 6460	120	13	3280– 4210	86	10				2420– 2770	58	8	3420– 4570	53	8
7270– 7970	131	14	4210– 5350	96	11	3650– 4480	82	10	3660– 4450	66	9	5650– 7990	60	9
			5350– 5750	105	12	4920– 5950	91	11						
8730– 9850	142	15	6820– 7550	115	13	6630– 7880	100	12	5500– 7130	74	10	9290– 14000	67	10
10500– 12200	153	16	8680– 9900	125	14	8910– 10400	109	13	8230– 11400	82	11	15200– 24300	74	11
12600– 15000	164	17				11900– 13700	118	14	12300– 17900	90	12	24300– 27300	80	12
15000– 18100	175	18	11000– 13000	135	15				17900– 20300	97	13	39700– 46700	87	13
18100– 21600	186	19	14000– 17000	145	16	16000– 18000	127	15	26600– 32000	105	14	64500– 79700	94	14
			17700– 22400	155	17	21400– 23700	136	16						
21600– 22700	196	20	22400– 28400	165	18	28500– 31100	145	17	39400– 50500	113	15	105000–136000	101	15
25900– 27900	207	21	28400– 29600	174	19	38000– 40700	154	18	58200– 80000	121	16	169000–200000	108	16
30900– 34300	218	22				50900– 67500	164	19	85700–124000	129	17			
37000– 42200	229	23	35900– 38600	184	20				124000–139000	136	18			
44200– 52800	240	24	45300– 50200	194	21	67500– 89600	173	20	183000–200000	144	19			
			57200– 65300	204	22	89600–119000	182	21						
52800– 63200	251	25	72200– 84900	214	23	119000–157000	191	22						
63200– 75700	262	26	91000–111000	224	24	157000–200000	200	23						
75700– 90100	273	27												
90100– 94900	283	28	115000–144000	234	25									
108000–116000	294	29	144000–181000	244	26									
			181000–189000	253	27									
128000–143000	305	30												
153000–175000	316	31												
183000–200000	327	32												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 6.0\,\%$

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$			$p_2 = 30.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 2110	Accept		1– 784	Accept		1– 361	Accept		1– 103	Accept		1– 39	Accept	
2110– 2160	59	8				362– 435	18	3	104– 130	5	1			
2170– 2390	69	9	785– 872	33	5	436– 563	26	4	131– 161	11	2	40– 46	1	0
			917– 1020	42	6				208– 236	18	3	47– 63	5	1
2450– 2670	79	10	1110– 1190	51	7	564– 679	34	5	319– 465	26	4	97– 111	11	2
2770– 2980	89	11	1350– 1630	61	8	737– 788	42	6				175– 248	18	3
3160– 3340	99	12	1630– 1970	70	9	955– 1220	51	7	466– 608	33	5	294– 341	24	4
3600– 3730	109	13				1220– 1350	59	8	677– 787	40	6			
4110– 4680	120	14	1970– 2380	79	10	1580– 2000	68	9	975– 1360	48	7	475– 693	31	5
			2380– 2760	88	11				1360– 1890	55	8	739– 888	37	6
4680– 5330	130	15	2870– 3170	97	12	2000– 2260	76	10	1890– 2260	62	9	1160– 1750	44	7
5330– 6070	140	16	3470– 3640	106	13	2550– 3210	85	11				1750– 2180	50	8
6070– 6920	150	17	4190– 5010	116	14	3210– 3730	93	12	2690– 3700	70	10	2710– 4050	57	9
6920– 7880	160	18				4090– 5120	102	13	3700– 5060	77	11			
7880– 8960	170	19	5010– 6000	125	15	5120– 6070	110	14	5060– 6150	84	12	4050– 5230	63	10
			6000– 7160	134	16				7050– 7590	91	13	6220– 9220	70	11
8960– 10200	180	20	7160– 7980	143	17	6480– 8100	119	15	9720– 13200	99	14	9220– 12300	76	12
10200– 11200	190	21	8630– 9080	152	18	8100– 10100	127	16				13800– 15200	82	13
11600– 12500	200	22	10300– 12300	162	19	10100– 10800	135	17	13200– 16300	106	15	20700– 28700	89	14
13300– 13900	210	23				12700– 15900	144	18	18200– 19900	113	16			
15100– 15500	220	24	12300– 14700	171	20	15900– 17200	152	19	25100– 33900	121	17	30800– 35000	95	15

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n + 1,c)$ as optimum plan.

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Single Sampling Tables for $p_1 = 6.0\,\%$ (Continued)

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$			$p_2 = 30.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
17200– 19600	231	25	14700– 17500	180	21	20000– 24800	161	20	33900– 42500	128	18	46200– 67600	102	16
19600– 22200	241	26	17500– 19500	189	22	24800– 27500	169	21	46500– 51600	135	19	67600– 80200	108	17
22200– 25200	251	27	20900– 22100	198	23	31100– 38600	178	22				102000–149000	115	18
25200– 28600	261	28	25000– 29700	208	24	38600– 43600	186	23	63800– 86100	143	20	149000–183000	121	19
28600– 32500	271	29				48400– 60000	195	24	86100–109000	150	21			
			29700– 35300	217	25				117000–132000	157	22			
32500– 36800	281	30	35300– 41900	226	26	60000– 69200	203	25	161000–200000	165	23			
36800– 40600	291	31	41900– 46700	235	27	75000– 92900	212	26						
41900– 45000	301	32	50100– 52700	244	28	92900–109000	220	27						
47600– 49900	311	33	59700– 70800	254	29	116000–144000	229	28						
54100– 55400	321	34				144000–177000	237	29						
			70800– 83900	263	30									
61500– 69700	332	35	83900– 99300	272	31	177000–187000	245	30						
69700– 79000	342	36	99300–110000	281	32									
79000– 89400	352	37	118000–124000	290	33									
89400–101000	362	38	141000–167000	300	34									
101000–115000	372	39												
			167000–200000	309	35									
115000–130000	382	40												
130000–142000	392	41												
147000–157000	402	42												
167000–174000	412	43												
190000–193000	422	44												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

$p_1 = 7.0\%$

Single Sampling Tables for $p_1 = 7.0\%$

$p_2 = 15.0\%$ N	n	c	$p_2 = 17.5\%$ N	n	c	$p_2 = 20.0\%$ N	n	c	$p_2 = 25.0\%$ N	n	c	$p_2 = 30.0\%$ N	n	c
1– 1960	Accept		1– 668	Accept		1– 287	Accept		1– 73	Accept		1– 19	Accept	
						288– 335	17	3	74– 101	5	1			
1960– 2030	73	10	669– 706	31	5	336– 357	24	4	102– 129	11	2	20– 36	1	0
2030– 2220	82	11	707– 735	39	6				161– 211	18	3	37– 45	5	1
2220– 2260	91	12	823– 958	48	7	424– 462	32	5	236– 331	25	4	76– 88	11	2
2440– 2680	101	13	959– 1120	57	8	532– 591	40	6				135– 213	18	3
2680– 2780	110	14	1120– 1220	65	9	663– 746	48	7	332– 460	32	5	214– 324	24	4
						818– 934	56	8	461– 627	39	6			
2940– 3240	120	15	1300– 1520	74	10	1000– 1160	64	9	628– 837	46	7	325– 478	30	5
3240– 3430	129	16	1520– 1760	83	11				838– 892	52	8	479– 662	36	6
3560– 3920	139	17	1760– 1970	91	12	1220– 1430	72	10	1110– 1220	59	9	701– 912	42	7
3920– 4230	148	18	2040– 2360	100	13	1490– 1800	80	11				1010– 1240	48	8
4310– 4750	158	19	2360– 2450	108	14	1800– 2170	88	12	1470– 1660	66	10	1460– 1690	54	9
						2170– 2620	96	13	1930– 2250	73	11			
4750– 5210	167	20	2720– 3130	117	15	2620– 3150	104	14	2520– 3040	80	12	2080– 2280	60	10
5210– 5740	177	21	3130– 3610	126	16				3290– 4100	87	13	2980– 4180	67	11
5740– 6310	186	22	3610– 3820	134	17	3150– 3780	112	15	4280– 5550	94	14	4180– 5860	73	12
6310– 6920	196	23	4150– 4760	143	18	3780– 4540	120	16				5860– 8180	79	13
6920– 7620	205	24	4760– 5470	152	19	4540– 5430	128	17	5550– 7220	101	15	8180– 11400	85	14
						5430– 6490	136	18	7220– 9390	108	16			
7620– 8350	215	25	5470– 5880	160	20	6490– 7760	144	19	9390– 12200	115	17	11400– 15100	91	15
8350– 9170	224	26	6260– 7190	169	21				12200– 15800	122	18	16000– 20000	97	16
9170– 9420	233	27	7190– 8220	178	22	7760– 9260	152	20	15800– 20300	129	19	22500– 26400	103	17
10000– 11000	243	28	8220– 8990	186	23	9260– 11000	160	21				31500– 34900	109	18
11000– 11500	252	29	9380– 10800	195	24	11000– 13100	168	22	20300– 21500	135	20	44400– 61500	116	19
						13100– 15600	176	23	26100– 28500	142	21			
12100– 13300	262	30	10800– 12300	204	25	15600– 18600	184	24	33500– 37800	149	22	61500– 85100	122	20
13300– 14000	271	31	12300– 13700	212	26				43100– 50200	156	23	85100–117000	128	21
14500– 15900	281	32	14000– 16000	221	27	18600– 22100	192	25	55300– 66700	163	24	117000–162000	134	22
15900– 17100	290	33	16000– 16400	229	28	22100– 26300	200	26				162000–200000	140	23
17400– 19100	300	34	18200– 20700	238	29	26300– 31200	208	27	70900– 90500	170	25			

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

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Single Sampling Tables for $p_1 = 7.0\,\%$ (Continued)

$p_2 = 15.0\,\%$			$p_2 = 17.5\,\%$			$p_2 = 20.0\,\%$			$p_2 = 25.0\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c
19100– 20900	309	35	20700– 23800	247	30	31200– 37000	216	28	90500–117000	177	26
20900– 22900	319	36	23800– 24800	255	31	37000– 43800	224	29	117000–150000	184	27
22900– 25100	328	37	27000– 30800	264	32				150000–200000	191	28
25100– 27500	338	38	30800– 35100	273	33	43800– 45200	231	30			
27500– 30100	347	39	35100– 37200	281	34	51900– 54000	239	31			
						61400– 64600	247	32			
30100– 32900	357	40	39900– 45500	290	35	72700– 77100	255	33			
32900– 36000	366	41	45500– 51800	299	36	86000– 92000	263	34			
36000– 39300	376	42	51800– 55700	307	37						
39300– 43000	385	43	58700– 67000	316	38	102000–110000	271	35			
43000– 44100	394	44	67000– 76100	325	39	120000–131000	279	36			
						142000–156000	287	37			
47000– 51500	404	45	76100– 83300	333	40	168000–186000	295	38			
51500– 53400	413	46	86300– 98600	342	41						
56200– 61500	423	47	98600–112000	351	42						
61500– 64700	432	48	112000–126000	359	43						
67100– 73400	442	49	126000–144000	368	44						
73400– 78300	451	50	144000–148000	376	45						
80000– 87600	461	51	164000–185000	385	46						
87600– 95400	470	52	185000–200000	394	47						
95400–104000	480	53									
104000–114000	489	54									
114000–125000	499	55									
125000–136000	508	56									
136000–148000	518	57									
148000–162000	527	58									
162000–177000	537	59									
177000–193000	546	60									
193000–197000	555	61									

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+1,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.20\,\%$

$p_2 = 1.50\,\%$			$p_2 = 1.75\,\%$			$p_2 = 2.00\,\%$			$p_2 = 2.50\,\%$			$p_2 = 3.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 15700	Accept		1– 7060	Accept		1– 3630	Accept		1– 1510	Accept		1– 508	Accept	
15700– 16100	155	2				3630– 3770	60	1				509– 728	5	0
			7060– 7200	155	2	4210– 4770	70	1	1510– 1640	60	1			
16100– 16600	275	3	7710– 8320	165	2				1810– 2020	70	1	943– 967	55	1
17600– 18700	285	3	9060– 9950	175	2	4770– 5140	160	2	2290– 2640	80	1	1070– 1200	65	1
20000– 21600	295	3	11100– 11400	185	2	5550– 6040	170	2	3100– 3440	90	1	1360– 1560	75	1
23500– 25400	305	3				6620– 7320	180	2				1820– 2170	85	1
			11400– 11900	285	3	8190– 9270	190	2	3440– 3710	160	2	2640– 2960	95	1
25400– 26300	425	4	12800– 13800	295	3				4090– 4550	170	2			
27900– 29800	435	4	15000– 16300	305	3	9670– 10500	285	3	5100– 5780	180	2	2960– 3250	155	2
32000– 34500	445	4	17900– 19900	315	3	11400– 12400	295	3	6630– 7720	190	2	3670– 4190	165	2
37500– 41000	455	4				13700– 15100	305	3				4820– 5620	175	2
			21100– 22700	425	4	16900– 19200	315	3	8500– 9260	270	3	6670– 8250	185	2
42500– 44900	580	5	24500– 26600	435	4				10300– 11500	280	3			
48000– 51500	590	5	29000– 31800	445	4	19800– 20700	410	4	13000– 14800	290	3	8250– 8710	250	3
55400– 60000	600	5	35100– 39400	455	4	22600– 24700	420	4	17100– 20100	300	3	9860– 11300	260	3
65400– 71800	610	5				27200– 30100	430	4				13000– 15100	270	3
			39400– 40000	560	5	33600– 37900	440	4	20100– 22200	380	4	17900– 21900	280	3
71800– 77100	735	6	43100– 46600	570	5				24800– 27800	390	4			
82600– 88800	745	6	50700– 55400	580	5	40200– 44000	540	5	31500– 36000	400	4	21900– 25200	350	4
95900–104000	755	6	60900– 67500	590	5	48100– 52900	550	5	41600– 46500	410	4	28800– 33200	360	4
114000–122000	765	6				58500– 65100	560	5				38700– 45700	370	4
			73100– 75000	700	6	73200– 80500	570	5	46500– 52000	490	5			
122000–124000	885	7	81000– 87800	710	6				58100– 65500	500	5	56200– 62800	445	5
132000–142000	895	7	95700–105000	720	6	80500– 84500	665	6	74300– 85200	510	5	71800– 82600	455	5
153000–165000	905	7	116000–128000	730	6	92400–101000	675	6	99000–106000	520	5	96200–113000	465	5
179000–196000	915	7				112000–124000	685	6				136000–143000	475	5
			135000–140000	840	7	139000–160000	695	6	106000–120000	600	6			
			151000–164000	850	7				135000–152000	610	6	143000–154000	540	6
			179000–200000	860	7	160000–176000	795	7	173000–200000	620	6	176000–200000	550	6
						193000–200000	805	7						

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.20\,\%$

$p_2 = 3.50\,\%$			$p_2 = 4.00\,\%$			$p_2 = 5.00\,\%$			$p_2 = 6.00\,\%$			$p_2 = 7.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 252	Accept		1– 156	Accept		1– 79	Accept		1– 48	Accept		1– 32	Accept	
253– 348	5	0	157– 218	5	0									
494– 740	15	0	309– 447	15	0	80– 118	5	0	49– 78	5	0	33– 58	5	0
						174– 256	15	0	123– 188	15	0	97– 156	15	0
741– 840	60	1	640– 730	60	1	393– 533	25	0	299– 468	25	0	259– 428	25	0
963– 1110	70	1	857– 1020	70	1									
1310– 1560	80	1	1230– 1510	80	1	534– 666	60	1	469– 554	55	1	429– 465	50	1
1900– 2380	90	1	1900– 2580	90	1	825– 1040	70	1	716– 941	65	1	625– 857	60	1
						1340– 1780	80	1	1270– 1800	75	1	1220– 1820	70	1
2730– 3230	150	2	2580– 2900	140	2									
3750– 4400	160	2	3420– 4100	150	2	2470– 2700	125	2	2450– 2950	115	2	2470– 3020	105	2
5240– 6360	170	2	4980– 6170	160	2	3330– 4180	135	2	3840– 5100	125	2	4110– 5780	115	2
7900– 8400	180	2	7860– 8710	170	2	5340– 7040	145	2	7010– 10600	135	2	8470– 11900	125	2
8400– 9460	235	3	8710– 10100	220	3	9580– 11900	195	3	10600– 14100	175	3	11900– 13200	155	3
11000– 12900	245	3	12000– 14500	230	3	14900– 19000	205	3	18600– 25000	185	3	17900– 24900	165	3
15400– 18600	255	3	17800– 22300	240	3	24900– 34800	215	3	35000– 44000	195	3	36000– 54600	175	3
23000– 24400	265	3												
			27600– 33800	300	4	34800– 40900	260	4	44000– 49900	230	4	54600– 74700	210	4
24400– 26400	320	4	40500– 49200	310	4	51000– 64700	270	4	65000– 86400	240	4	103000–147000	220	4
30700– 36100	330	4	60800– 76900	320	4	84000–113000	280	4	118000–175000	250	4			
42900– 51800	340	4												
63800– 69100	350	4	84500– 92700	375	5	124000–137000	325	5	175000–200000	290	5			
			110000–133000	385	5	170000–200000	335	5						
69100– 72000	405	5	163000–200000	395	5									
83700– 98200	415	5												
116000–140000	425	5												
172000–192000	435	5												
192000–200000	495	6												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.25\,\%$

$p_2 = 1.50\,\%$			$p_2 = 1.75\,\%$			$p_2 = 2.00\,\%$			$p_2 = 2.50\,\%$			$p_2 = 3.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 17100	Accept		1– 7520	Accept		1– 4100	Accept		1– 1510	Accept		1– 586	Accept	
17100– 18600	250	3	7520– 7670	140	2	4100– 4290	55	1				587– 863	5	0
			8380– 9380	150	2				1510– 1610	55	1			
20200– 21400	375	4				4290– 4480	140	2	1800– 2040	65	1	864– 884	50	1
22900– 24700	385	4	9380– 9550	250	3	4840– 5280	150	2	2370– 2830	75	1	983– 1110	60	1
26700– 29200	395	4	10200– 11100	260	3	5820– 6500	160	2				1260– 1470	70	1
			12000– 13200	270	3				2830– 3040	145	2	1740– 2130	80	1
29800– 31300	515	5	14600– 15300	280	3	7490– 7920	255	3	3370– 3760	155	2			
33600– 36200	525	5				8620– 9450	265	3	4250– 4860	165	2	2350– 2430	140	2
39200– 42700	535	5	15300– 16200	380	4	10400– 11700	275	3	5660– 6200	175	2	2730– 3110	150	2
			17500– 19100	390	4	13200– 13700	285	3				3590– 4190	160	2
45000– 46100	655	6	20900– 23100	400	4				6200– 6420	245	3	5000– 5840	170	2
49400– 53200	665	6				13700– 15000	375	4	7130– 7980	255	3			
57600– 62700	675	6	25500– 27600	510	5	16500– 18200	385	4	9010– 10300	265	3	5840– 6710	235	3
			30000– 32700	520	5	20200– 22700	395	4	12000– 13000	275	3	7680– 8880	245	3
68600– 72700	800	7	36000– 39900	530	5							10400– 12500	255	3
78200– 84500	810	7				24700– 25700	490	5	13000– 14400	350	4			
91900–100000	820	7	42400– 43100	635	6	28100– 30900	500	5	16100– 18200	360	4	13700– 15700	325	4
			46700– 50800	645	6	34300– 38400	510	5	20700– 24000	370	4	18000– 20900	335	4
105000–107000	940	8	55700– 61400	655	6							24500– 29400	345	4
115000–124000	950	8	68300– 70300	665	6	44200– 47400	610	6	26700– 28300	450	5			
134000–146000	960	8				52000– 57500	620	6	31600– 35600	460	5	31000– 35600	415	5
			70300– 72400	765	7	64000– 72000	630	6	40600– 46800	470	5	41000– 47600	425	5
160000–168000	1085	9	78600– 85700	775	7							56200– 69200	435	5
181000–196000	1095	9	94000–104000	785	7	78800– 86800	730	7	54100– 61100	555	6			
						95600–106000	740	7	68800– 78200	565	6	69200– 79800	505	6
			116000–121000	895	8	119000–134000	750	7	89800–104000	575	6	91900–107000	515	6
			132000–144000	905	8							126000–153000	525	6
			158000–175000	915	8	139000–144000	845	8	108000–117000	655	7			
						158000–175000	855	8	132000–149000	665	7	153000–177000	595	7
			191000–200000	1025	9	194000–200000	865	8	170000–200000	675	7			

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.25\,\%$

$p_2 = 3.50\,\%$			$p_2 = 4.00\,\%$			$p_2 = 5.00\,\%$			$p_2 = 6.00\,\%$			$p_2 = 7.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 259	Accept		1– 154	Accept		1– 75	Accept		1– 45	Accept		1– 30	Accept	
260– 370	5	0	155– 220	5	0									
554– 657	15	0	320– 486	15	0	76– 114	5	0	46– 74	5	0	31– 55	5	0
						171– 258	15	0	119– 185	15	0	93– 151	15	0
658– 736	55	1	556– 618	55	1	414– 453	25	0	304– 404	25	0	260– 365	25	0
846– 984	65	1	728– 866	65	1									
1170– 1410	75	1	1050– 1310	75	1	454– 533	55	1	405– 542	55	1	366– 451	50	1
1750– 2120	85	1	1680– 2000	85	1	660– 831	65	1	712– 960	65	1	615– 864	60	1
						1080– 1440	75	1	1360– 1830	75	1	1280– 1820	70	1
2120– 2210	135	2	2000– 2160	130	2									
2550– 2980	145	2	2560– 3070	140	2	1870– 2230	120	2	1830– 2310	110	2	1820– 2230	100	2
3530– 4260	155	2	3750– 4700	150	2	2780– 3550	130	2	3030– 4100	120	2	3060– 4360	110	2
5270– 5790	165	2				4660– 6290	140	2	5800– 6920	130	2	6570– 7600	120	2
			5880– 6570	205	3									
5790– 6690	220	3	7830– 9480	215	3	6290– 6640	180	3	6920– 8730	165	3	7600– 9960	150	3
7820– 9270	230	3	11700– 14800	225	3	8230– 10400	190	3	11500– 15700	175	3	13800– 19900	160	3
11200– 13800	240	3				13500– 18100	200	3	22300– 24600	185	3			
			16300– 19100	280	4							30000– 42800	200	4
14800– 16600	300	4	22900– 27900	290	4	20000– 23300	245	4	24600– 31700	220	4	60000– 87800	210	4
19500– 23100	310	4	34800– 43700	300	4	29200– 37400	255	4	42000– 57300	230	4			
27800– 34300	320	4				49400– 62100	265	4				115000–131000	245	5
			43700– 45500	350	5				84900–112000	275	5	180000–200000	255	5
36700– 40300	380	5	54200– 65500	360	5	62100– 80400	310	5	149000–200000	285	5			
47200– 55900	390	5	80400–101000	370	5	102000–132000	320	5						
67400– 82800	400	5												
			116000–127000	425	6	186000–200000	370	6						
89700– 96200	460	6	152000–184000	435	6									
113000–133000	470	6												
160000–200000	480	6												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.30\,\%$

$p_2 = 1.50\,\%$			$p_2 = 1.75\,\%$			$p_2 = 2.00\,\%$			$p_2 = 2.50\,\%$			$p_2 = 3.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 18700	Accept		1– 8220	Accept		1– 4270	Accept		1– 1520	Accept		1– 675	Accept	
			8220– 8270	125	2							676– 812	5	0
18700– 19500	335	4				4270– 4510	130	2	1520– 1570	50	1			
21000– 22900	345	4	8360– 8630	225	3	4940– 5500	140	2	1760– 2040	60	1	813– 889	50	1
			9310– 10100	235	3							1000– 1150	60	1
23600– 24000	460	5	11100– 12200	245	3	6260– 6920	235	3	2450– 2710	135	2	1350– 1620	70	1
25700– 27700	470	5				7590– 8400	245	3	3020– 3410	145	2			
30100– 32700	480	5	12200– 13000	345	4	9420– 10400	255	3	3900– 4540	155	2	1970– 2260	135	2
			14100– 15400	355	4							2580– 2990	145	2
32700– 34400	595	6	17000– 18600	365	4	10400– 11500	345	4	4880– 5340	230	3	3530– 4260	155	2
37000– 40100	605	6				12600– 14000	355	4	5970– 6760	240	3			
43800– 45900	615	6	18600– 19700	465	5	15700– 17200	365	4	7750– 9040	250	3	4450– 5070	220	3
			21500– 23500	475	5							5810– 6770	230	3
45900– 49600	730	7	26000– 28500	485	5	17200– 18700	455	5	9360– 10000	325	4	8010– 9470	240	3
53600– 58300	740	7				20600– 22900	465	5	11300– 12700	335	4			
			28500– 29800	585	6	25700– 28300	475	5	14600– 17000	345	4	9470– 10800	305	4
64500– 66500	860	8	32400– 35500	595	6							12400– 14500	315	4
71600– 77600	870	8	39200– 43500	605	6	28300– 30200	565	6	17500– 18400	420	5	17200– 19600	325	4
84700– 91200	880	8				33300– 36900	575	6	20600– 23400	430	5			
			43500– 44700	705	7	41400– 46300	585	6	26800– 31200	440	5	19600– 22300	390	5
91200– 95800	995	9	48700– 53300	715	7							25700– 30100	400	5
104000–112000	1005	9	58800– 66300	725	7	46300– 48200	675	7	32400– 37200	520	6	35800– 39700	410	5
123000–129000	1015	9				53100– 59000	685	7	42100– 48200	530	6			
			66300– 72700	830	8	66000– 75300	695	7	56000– 59200	540	6	39700– 45300	475	6
129000–138000	1130	10	79500– 87700	840	8							52400– 61400	485	6
149000–163000	1140	10	97600–101000	850	8	75300– 84300	790	8	59200– 66300	615	7	73300– 79800	495	6
						93500–104000	800	8	75000– 85800	625	7			
182000–200000	1265	11	101000–108000	950	9	118000–122000	810	8	99400–107000	635	7	79800– 91200	560	7
			118000–130000	960	9							106000–124000	570	7
			145000–153000	970	9	122000–133000	900	9	107000–117000	710	8	148000–159000	580	7
						147000–164000	910	9	133000–151000	720	8			
			153000–160000	1070	10	185000–196000	920	9	175000–193000	730	8	159000–182000	645	8
			175000–193000	1080	10									
						196000–200000	1010	10	193000–200000	805	9			

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.30\,\%$

$p_2 = 3.50\,\%$			$p_2 = 4.00\,\%$			$p_2 = 5.00\,\%$			$p_2 = 6.00\,\%$			$p_2 = 7.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 262	Accept		1– 149	Accept		1– 70	Accept		1– 41	Accept		1– 27	Accept	
263– 386	5	0												
			150– 217	5	0	71– 109	5	0	42– 70	5	0	28– 51	5	0
590– 638	50	1	326– 492	15	0	166– 256	15	0	113– 179	15	0	88– 145	15	0
733– 853	60	1							303– 348	25	0	256– 321	25	0
1010– 1230	70	1	493– 519	50	1	398– 423	50	1						
1550– 1750	80	1	609– 724	60	1	521– 653	60	1	349– 408	50	1	322– 436	50	1
			881– 1100	70	1	842– 1120	70	1	530– 707	60	1	605– 874	60	1
1750– 1980	130	2	1420– 1620	80	1				986– 1440	70	1			
2310– 2740	140	2				1500– 1830	115	2				1410– 1640	95	2
3310– 4120	150	2	1620– 1870	125	2	2290– 2960	125	2	1440– 1790	105	2	2240– 3190	105	2
			2240– 2720	135	2	3970– 4550	135	2	2350– 3210	115	2	4850– 5320	115	2
4320– 4620	205	3	3400– 4320	145	2									
5390– 6380	215	3				4550– 5570	175	3	4820– 5320	155	3	5320– 7480	145	3
7690– 9520	225	3	4320– 4970	195	3	7040– 9150	185	3	6960– 9380	165	3	10600– 15700	155	3
			5960– 7290	205	3				13200– 15500	175	3			
10000– 11900	285	4	9160– 10800	215	3	12900– 16200	235	4				18600– 24000	190	4
14100– 16900	295	4				20600– 27000	245	4	15500– 19800	210	4	33500– 49200	200	4
20800– 22500	305	4	10800– 12500	265	4				26400– 36500	220	4			
			15100– 18500	275	4	35700– 46200	295	5				63400– 75300	235	5
22500– 25500	360	5	23400– 26200	285	4	59000– 77700	305	5	47900– 55000	260	5	104000–151000	245	5
30100– 36100	370	5							72600– 98900	270	5			
44200– 49700	380	5	26200– 30700	335	5	96700–103000	350	6						
			37200– 45900	345	5	130000–166000	360	6	145000–200000	315	6			
49700– 54000	435	6	58100– 62400	355	5									
63500– 75900	445	6												
92400–108000	455	6	62400– 74400	405	6									
			90200–112000	415	6									
108000–113000	510	7												
133000–158000	520	7	146000–178000	475	7									
191000–200000	530	7												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.35\,\%$

$p_2 = 1.50\,\%$			$p_2 = 1.75\,\%$			$p_2 = 2.00\,\%$			$p_2 = 2.50\,\%$			$p_2 = 3.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 20000	Accept		1– 8500	Accept		1– 4350	Accept		1– 1520	Accept		1– 761	Accept	
20000– 20400	305	4	8500– 8990	210	3	4350– 4510	120	2	1520– 1700	50	1			
20500– 21900	420	5	9880– 10500	220	3	5020– 5480	130	2	1980– 2180	60	1	762– 794	45	1
23700– 26200	430	5										893– 1020	55	1
			10500– 11000	315	4	5480– 5900	215	3	2180– 2370	125	2	1200– 1460	65	1
26200– 28300	545	6	11900– 13100	325	4	6480– 7200	225	3	2650– 3000	135	2			
30700– 33500	555	6	14800– 15900	430	5	8110– 8430	235	3	3460– 4020	145	2	1700– 1830	125	2
34300– 36900	670	7	17400– 19200	440	5							2090– 2410	135	2
40000– 43800	680	7				8430– 9270	320	4	4020– 4340	215	3	2850– 3440	145	2
			21100– 23200	545	6	10200– 11400	330	4	4870– 5530	225	3			
45400– 48300	795	8	25400– 28100	555	6	13000– 14400	425	5	6390– 7190	235	3	3560– 3730	205	3
52400– 57200	805	8	30300– 33600	660	7	16000– 17900	435	5				4270– 4940	215	3
60200– 63200	920	9	36900– 41000	670	7				7190– 7590	305	4	5830– 7050	225	3
68500– 74800	930	9				20000– 22100	530	6	8520– 9690	315	4			
			43200– 44300	770	8	24500– 27500	540	6	11200– 12600	325	4	7050– 8230	290	4
80000– 82600	1045	10	48500– 53400	780	8	30600– 33700	635	7				9550– 11300	300	4
89500– 97600	1055	10	59400– 61600	790	8	37400– 41900	645	7	12600– 12900	395	5	13500– 15400	370	5
106000–117000	1175	11							14500– 16500	405	5	17900– 21100	380	5
127000–141000	1185	11	61600– 63700	885	9	46600– 50900	740	8	19000– 21600	415	5			
			69700– 76900	895	9	56600– 63500	750	8				25400– 28400	450	6
141000–152000	1300	12	85700– 87700	905	9	70500– 76600	845	9	21600– 24200	490	6	32900– 38800	460	6
166000–182000	1310	12				85100– 95600	855	9	27500– 31700	500	6	47100– 51600	530	7
188000–200000	1425	13	87700– 91200	1000	10				36600– 40100	580	7	59800– 70400	540	7
			100000–110000	1010	10	106000–115000	950	10	45500– 52200	590	7			
			124000–130000	1115	11	128000–143000	960	10				86600– 93100	610	8
			143000–158000	1125	11	160000–171000	1055	11	61700– 65800	670	8	108000–127000	620	8
						190000–200000	1065	11	74500– 85400	680	8	152000–159000	630	8
			176000–186000	1230	12				99300–104000	690	8			
												159000–167000	690	9
									104000–107000	760	9	193000–200000	700	9
									121000–139000	770	9			
									161000–173000	780	9			
									173000–200000	855	10			

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.40\,\%$

$p_2 = 1.50\,\%$

N	n	c
1– 21100	Accept	
21100– 22500	390	5
22500– 24200	500	6
26300– 27800	510	6
27800– 30500	620	7
33300– 34800	630	7
34800– 35800	735	8
38800– 42400	745	8
44000– 45500	855	9
49400– 54100	865	9
55600– 57900	975	10
63000– 69100	985	10
70500– 73600	1095	11
80200– 89300	1105	11
89300– 93700	1215	12
102000–113000	1225	12
113000–119000	1335	13
130000–144000	1345	13
144000–151000	1455	14
165000–182000	1465	14
182000–192000	1575	15

$p_2 = 1.75\,\%$

N	n	c
1– 8820	Accept	
8820– 9320	195	3
9320– 9710	290	4
10600– 11700	300	4
12400– 13500	400	5
14800– 16700	410	5
16700– 17200	505	6
18800– 20800	515	6
22700– 23900	615	7
26200– 29100	625	7
30800– 33100	725	8
36500– 40600	735	8
41800– 45800	835	9
50600– 56500	845	9
56500– 63200	945	10
70000– 76300	955	10
76300– 79100	1050	11
87000– 96700	1060	11
103000–108000	1160	12
120000–133000	1170	12
138000–149000	1270	13
164000–186000	1280	13
186000–200000	1380	14

$p_2 = 2.00\,\%$

N	n	c
1– 4450	Accept	
4450– 4940	115	2
4940– 5360	200	3
5920– 6630	210	3
7130– 7970	300	4
8850– 9970	310	4
10400– 10600	395	5
11700– 13100	405	5
14800– 15200	415	5
15200– 17100	500	6
19100– 22000	510	6
22000– 24700	600	7
27600– 31700	610	7
31700– 35400	700	8
39600– 45400	710	8
45400– 50500	800	9
56500– 64900	810	9
64900– 71700	900	10
80300– 92500	910	10
92500–102000	1000	11
114000–131000	1010	11
131000–143000	1100	12
160000–181000	1110	12
187000–200000	1200	13

$p_2 = 2.50\,\%$

N	n	c
1– 1510	Accept	
1510– 1590	45	1
1860– 1980	55	1
1980– 2040	115	2
2270– 2560	125	2
2950– 3420	135	2
3420– 3850	205	3
4350– 5010	215	3
5780– 6230	290	4
7040– 8070	300	4
9520– 9810	375	5
11100– 12600	385	5
14700– 15500	395	5
15500– 17100	465	6
19400– 22500	475	6
24800– 26100	550	7
29600– 34000	560	7
39400– 44700	640	8
51200– 59500	650	8
62200– 67100	725	9
76600– 88500	735	9
98000–114000	815	10
131000–153000	825	10
153000–170000	900	11
194000–200000	910	11

$p_2 = 3.00\,\%$

N	n	c
1– 722	Accept	
723– 781	45	1
892– 1040	55	1
1260– 1490	65	1
1490– 1650	120	2
1890– 2210	130	2
2650– 2960	140	2
2960– 3060	195	3
3510– 4080	205	3
4840– 5510	215	3
5510– 6110	275	4
7070– 8350	285	4
9910– 10300	350	5
11900– 14000	360	5
16700– 17500	370	5
17500– 19700	430	6
23000– 27300	440	6
30600– 32200	505	7
37400– 44100	515	7
52800– 60500	585	8
70900– 84700	595	8
90900– 97500	660	9
114000–134000	670	9
156000–181000	740	10

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.35\,\%$

$p_2 = 3.50\,\%$			$p_2 = 4.00\,\%$			$p_2 = 5.00\,\%$			$p_2 = 6.00\,\%$			$p_2 = 7.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 259	Accept		1– 142	Accept		1– 65	Accept		1– 37	Accept		1– 24	Accept	
260– 394	5	0	143– 211	5	0	66– 102	5	0	38– 65	5	0			
			324– 446	15	0	158– 248	15	0	106– 171	15	0	25– 47	5	0
543– 624	50	1										82– 137	15	0
727– 861	60	1	447– 502	50	1	357– 406	50	1	308– 391	50	1	248– 281	25	0
1050– 1320	70	1	596– 720	60	1	506– 644	60	1	516– 702	60	1			
			895– 1150	70	1	851– 1230	70	1	1010– 1190	70	1	282– 310	45	1
1480– 1750	125	2										421– 591	55	1
2060– 2470	135	2	1360– 1600	120	2	1230– 1470	110	2	1190– 1360	100	2	884– 1170	65	1
3050– 3400	145	2	1920– 2370	130	2	1850– 2400	120	2	1790– 2440	110	2			
			3010– 3360	140	2	3260– 3450	130	2	3620– 4150	150	3	1170– 1620	95	2
3400– 3630	195	3							5480– 7500	160	3	2280– 3430	105	2
4250– 5060	205	3	3360– 3690	185	3	3450– 3710	165	3				3950– 5540	140	3
6160– 7300	215	3	4420– 5420	195	3	4640– 5970	175	3	10500– 12100	200	4	7880– 12500	150	3
			6830– 7770	205	3	7980– 9010	185	3	16000– 22000	210	4			
7300– 8290	270	4							29800– 34500	250	5	12500– 18200	185	4
9800– 11800	280	4	7770– 9610	255	4	9010– 11100	225	4	45800– 63300	260	5	26200– 38500	195	4
			11700– 14700	265	4	14100– 18700	235	4				38500– 42400	225	5
15100– 15700	340	5	17300– 20300	320	5	22700– 25800	280	5	82500– 96800	300	6	58700– 84800	235	5
18500– 22100	350	5	24700– 30800	330	5	32700– 42700	290	5	129000–179000	310	6			
26900– 30800	360	5										118000–134000	270	6
			37600– 42200	385	6	56200– 59300	335	6				186000–200000	280	6
30800– 34300	415	6	51200– 63500	395	6	74800– 96600	345	6						
40700– 49200	425	6				130000–138000	355	6						
61900– 74500	490	7	81200– 87000	450	7									
89100–109000	500	7	105000–129000	460	7	138000–169000	395	7						
			164000–175000	470	7									
124000–136000	560	8												
161000–200000	570	8	175000–200000	520	8									

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.40\,\%$

$p_2 = 3.50\,\%$			$p_2 = 4.00\,\%$			$p_2 = 5.00\,\%$			$p_2 = 6.00\,\%$			$p_2 = 7.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 249	Accept		1– 133	Accept		1– 59	Accept		1– 33	Accept		1– 21	Accept	
250– 389	5	0	134– 199	5	0	60– 94	5	0	34– 59	5	0	22– 43	5	0
			312– 406	15	0	147– 235	15	0	98– 159	15	0	76– 128	15	0
497– 526	45	1												
608– 714	55	1	407– 482	50	1	324– 387	50	1	281– 373	50	1	249– 293	45	1
864– 1080	65	1	579– 710	60	1	488– 630	60	1	497– 691	60	1	402– 575	55	1
			903– 1170	70	1	854– 1050	70	1	1000– 1340	100	2	889– 972	65	1
1290– 1520	120	2							1800– 2570	110	2			
1800– 2180	130	2	1170– 1350	115	2	1050– 1170	105	2				973– 1150	90	2
2770– 3270	190	3	1620– 2000	125	2	1460– 1890	115	2	2860– 3190	145	3	1600– 2350	100	2
3890– 4730	200	3	2560– 2720	135	2	2570– 2730	125	2	4220– 5830	155	3	3050– 4030	135	3
									7680– 9530	195	4	5730– 8830	145	3
5600– 6610	260	4	2720– 3200	180	3	2730– 3020	160	3	12900– 18200	205	4			
7880– 9640	270	4	3890– 4860	190	3	3800– 4950	170	3				8830– 9820	175	4
10900– 13000	330	5	5860– 7200	245	4	6600– 7400	215	4	20100– 28000	245	5	13600– 20000	185	4
15500– 19100	340	5	8840– 11200	255	4	9390– 12300	225	4	38600– 51000	255	5	25500– 32200	220	5
									51000– 60900	290	6	45700– 71500	230	5
20700– 25000	400	6	12100– 13000	305	5	15500– 17700	270	5	81900–115000	300	6			
30000– 37000	410	6	15800– 19600	315	5	22600– 29800	280	5				71500–105000	265	6
39000– 47600	470	7	24700– 28000	370	6	35800– 41700	325	6	130000–174000	340	7	153000–200000	275	6
57400– 72300	480	7	34100– 42700	380	6	53500– 71000	335	6						
72300– 75900	535	8	50000– 59500	435	7	81500– 97500	380	7						
90000–109000	545	8	73200– 92500	445	7	125000–167000	390	7						
134000–142000	605	9	100000–126000	500	8	184000–200000	435	8						
169000–200000	615	9	156000–200000	510	8									

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.50\,\%$

$p_2 = 1.50\,\%$			$p_2 = 1.75\,\%$			$p_2 = 2.00\,\%$			$p_2 = 2.50\,\%$			$p_2 = 3.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 22700	Accept		1– 9030	Accept		1– 4420	Accept		1– 1410	Accept		1– 625	Accept	
22700– 23300	535	7	9030– 9630	260	4	4420– 4950	180	3	1410– 1460	40	1	626– 725	45	1
24700– 27400	645	8	9630– 10300	350	5							851– 1030	55	1
			11300– 11900	360	5	5500– 5830	265	4	1650– 1720	105	2			
28700– 29700	750	9				6490– 7370	275	4	1930– 2210	115	2	1190– 1260	110	2
32500– 33700	760	9	11900– 13000	450	6	7370– 7610	355	5	2610– 2780	185	3	1450– 1700	120	2
33700– 35300	860	10	14400– 14900	460	6	8450– 9530	365	5	3150– 3640	195	3	2060– 2170	130	2
38700– 39700	870	10	14900– 16400	550	7									
			18200– 18700	560	7	9900– 10900	450	6	4060– 4310	265	4	2170– 2510	185	3
39700– 42000	970	11				12200– 13200	460	6	4900– 5670	275	4	2940– 3540	195	3
46800– 50000	1080	12	18700– 20700	650	8	13200– 14000	540	7	6150– 6490	345	5	3690– 4080	255	4
55300– 59600	1190	13	23500– 26100	750	9	15600– 17600	550	7	7390– 8570	355	5	4760– 5690	265	4
65400– 70900	1300	14	29500– 32800	850	10									
			36900– 41200	950	11	17600– 19800	635	8	9140– 9580	425	6	6060– 6400	325	5
77300– 84500	1410	15				22300– 23300	645	8	10900– 12700	435	6	7450– 8860	335	5
91400–101000	1520	16	46200– 51500	1050	12	23300– 25000	725	9	13400– 14000	505	7	9750– 11400	400	6
108000–120000	1630	17	57600– 64400	1150	13	28100– 30800	735	9	15900– 18500	515	7	13500– 15400	410	6
128000–142000	1740	18	71900– 80200	1250	14									
			89500– 99800	1350	15	30800– 35200	820	10	19500– 20100	585	8	15400– 17300	470	7
151000–154000	1845	19				40400– 43900	910	11	23000– 26800	595	8	20400– 24100	480	7
169000–178000	1855	19	111000–124000	1450	16	49500– 53000	920	11	28200– 33000	670	9	24100– 25900	540	8
178000–183000	1955	20	138000–154000	1550	17				38400– 40500	680	9	30400– 37600	550	8
			172000–191000	1650	18	53000– 54900	1000	12						
						61500– 69400	1010	12	40500– 47200	750	10	37600– 45200	615	9
						69400– 76400	1095	13	54800– 58000	760	10	54000– 58400	625	9
						86400– 90600	1105	13	58000– 67000	830	11	58400– 66900	685	10
									77900– 82700	840	11	79400– 90000	695	10
						90600– 94900	1185	14						
						107000–118000	1195	14	82700– 95000	910	12	90000– 98800	755	11
						118000–132000	1280	15	110000–118000	920	12	117000–138000	765	11
						149000–154000	1290	15	118000–134000	990	13	138000–145000	825	12
									156000–167000	1000	13	171000–200000	835	12
						154000–163000	1370	16						
						184000–200000	1380	16	167000–189000	1070	14			

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.50\,\%$

$p_2 = 3.50\,\%$			$p_2 = 4.00\,\%$			$p_2 = 5.00\,\%$			$p_2 = 6.00\,\%$			$p_2 = 7.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 207	Accept		1– 106	Accept		1– 46	Accept		1– 25	Accept		1– 16	Accept	
208– 330	5	0	107– 163	5	0	47– 75	5	0	26– 47	5	0	17– 34	5	0
			259– 337	15	0	120– 193	15	0	80– 131	15	0	62– 105	15	0
417– 467	45	1										193– 211	25	0
550– 662	55	1	338– 359	45	1	271– 340	50	1	232– 250	45	1			
829– 1000	65	1	429– 523	55	1	436– 578	60	1	329– 445	55	1	212– 255	45	1
			662– 899	65	1	798– 874	100	2	643– 755	65	1	356– 523	55	1
1000– 1080	110	2				1100– 1440	110	2				737– 1060	90	2
1270– 1540	120	2	900– 1080	110	2				756– 946	95	2	1560– 2010	100	2
1970– 2120	175	3	1310– 1660	120	2	1870– 2390	155	3	1280– 1900	105	2			
2500– 3040	185	3	1910– 2280	170	3	3120– 4030	165	3	1900– 2360	140	3	2010– 2820	130	3
			2800– 3680	180	3	4030– 4900	205	4	3210– 4510	150	3	4130– 5180	140	3
3620– 3870	240	4				6370– 8370	215	4				5180– 7150	170	4
4600– 5610	250	4	3680– 4530	230	4				4510– 5610	185	4	10500– 12900	180	4
6370– 6850	305	5	5620– 6920	240	4	8370– 9780	255	5	7670– 10400	195	4			
8180– 10000	315	5	6920– 8730	290	5	12700– 17100	265	5	10400– 13000	230	5	12900– 17700	210	5
			10900– 12600	300	5	17100– 19200	305	6	17900– 23400	240	5	26000– 31900	220	5
10900– 11900	370	6				24800– 34400	315	6				31900– 43300	250	6
14300– 17600	380	6	12600– 13500	345	6				23400– 29800	275	6	63600– 77600	260	6
18500– 20400	435	7	16600– 20900	355	6	34400– 37300	355	7	41100– 52400	285	6			
24600– 31000	445	7	22700– 25200	405	7	48000– 64200	365	7	52400– 67500	320	7	77600–105000	290	7
			31100– 40500	415	7	69700– 92200	410	8	93500–116000	330	7	154000–187000	300	7
31000– 34800	500	8				123000–138000	420	8				187000–200000	330	8
42000– 51800	510	8	40500– 46800	465	8				116000–152000	365	8			
51800– 58900	565	9	58000– 72200	475	8	138000–176000	460	9						
71400– 86200	575	9	72200– 86300	525	9									
			108000–128000	535	9									
86200– 99300	630	10												
121000–143000	640	10	128000–159000	585	10									
143000–167000	695	11												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.60\,\%$

$p_2 = 1.50\,\%$			$p_2 = 1.75\,\%$			$p_2 = 2.00\,\%$			$p_2 = 2.50\,\%$			$p_2 = 3.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 23100	Accept		1– 8630	Accept		1– 4010	Accept		1– 1190	Accept		1– 493	Accept	
23100– 23800	670	9	8630– 9290	320	5	4010– 4420	165	3	1190– 1320	40	1	494– 524	40	1
			9290– 9710	405	6	4420– 4920	245	4	1360– 1440	100	2	606– 720	50	1
24500– 25900	770	10							1630– 1900	110	2	895– 952	60	1
27400– 28200	870	11	11000– 12100	500	7	5590– 6250	330	5						
30700– 33800	975	12	13000– 13700	590	8	7100– 7860	415	6	2050– 2230	175	3	953– 987	105	2
34500– 36900	1075	13	15500– 17100	685	9	8980– 9810	500	7	2550– 3010	185	3	1140– 1350	115	2
38800– 40300	1175	14							3010– 3300	250	4	1650– 1840	175	3
			18500– 19200	775	10	11300– 12100	585	8	3790– 4320	260	4	2160– 2640	185	3
43700– 49200	1280	15	21400– 22000	785	10	13700– 14200	595	8						
49200– 53000	1380	16	22000– 23900	870	11	14200– 15000	670	9	4320– 4720	325	5	2640– 2760	240	4
55500– 57900	1480	17				16900– 17700	680	9	5440– 6040	335	5	3220– 3860	250	4
62500– 70400	1585	18	26100– 26700	960	12				6040– 6620	400	6	4080– 4650	310	5
70400– 76100	1685	19	29700– 30900	970	12	17700– 18300	755	10	7650– 8340	410	6	5520– 6120	320	5
			30900– 33100	1055	13	20600– 22000	765	10						
79300– 82900	1785	20	36600– 41000	1150	14	22000– 25200	845	11	8340– 9140	475	7	6120– 6560	375	6
89400–101000	1890	21							10600– 11400	485	7	7730– 9040	385	6
101000–109000	1990	22	43200– 45400	1240	15	27300– 30500	930	12	11400– 12500	550	8	9040– 10700	445	7
113000–118000	2090	23	51000– 56100	1335	16	33700– 37000	1015	13	14500– 15400	560	8			
128000–143000	2195	24	60100– 62100	1425	17	41500– 44700	1100	14				13100– 14700	510	8
			70800– 76500	1520	18				15400– 16900	625	9	17500– 19100	520	8
143000–155000	2295	25	83500– 94400	1615	19	51100– 53900	1185	15	19700– 20700	635	9	19100– 20000	575	9
161000–168000	2395	26				61200– 62800	1195	15	20700– 22800	700	10	23600– 27500	585	9
182000–200000	2500	27	98100–104000	1705	20	62800– 65000	1270	16	26600– 27800	710	10			
			115000–128000	1800	21	73600– 77200	1280	16				27500– 31900	645	10
			135000–140000	1890	22				27800– 30700	775	11	39300– 43100	710	11
			159000–172000	1985	23	77200– 88300	1360	17	35800– 37100	785	11	51300– 56600	720	11
			187000–189000	2075	24	94600–106000	1445	18	37100– 41000	850	12			
						116000–127000	1530	19	47900– 49400	860	12	56600– 68600	780	12
												80400– 91600	845	13
						142000–152000	1615	20	49400– 54800	925	13	110000–114000	855	13
						173000–182000	1700	21	65500– 72900	1000	14			
									86900– 96800	1075	15	114000–122000	910	14
									115000–128000	1150	16	146000–163000	920	14
									152000–170000	1225	17	163000–193000	980	15

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.60\,\%$

$p_2 = 3.50\,\%$ N	n	c	$p_2 = 4.00\,\%$ N	n	c	$p_2 = 5.00\,\%$ N	n	c	$p_2 = 6.00\,\%$ N	n	c	$p_2 = 7.00\,\%$ N	n	c
1– 144	Accept		1– 74	Accept		1– 31	Accept		1– 16	Accept		1– 9	Accept	
145– 225	5	0	75– 115	5	0	32– 54	5	0	17– 34	5	0	10– 25	5	0
			181– 279	15	0	88– 139	15	0	60– 97	15	0	47– 80	15	0
340– 382	45	1							167– 202	25	0	142– 182	25	0
454– 552	55	1	280– 292	45	1	227– 280	50	1						
702– 800	65	1	352– 432	55	1	361– 483	60	1	203– 271	50	1	183– 211	45	1
			553– 714	65	1	633– 759	100	2	371– 541	60	1	295– 437	55	1
801– 960	110	2				981– 1360	110	2	597– 825	95	2	569– 662	85	2
1160– 1480	120	2	715– 786	105	2				1150– 1370	105	2	935– 1400	95	2
1480– 1700	170	3	957– 1200	115	2	1360– 1740	150	3						
2050– 2540	180	3	1420– 1780	165	3	2300– 2690	160	3	1370– 1620	135	3	1400– 1820	125	3
			2230– 2550	175	3	2690– 2950	195	4	2200– 2960	145	3	2650– 3320	135	3
2540– 2810	230	4				3810– 5100	205	4	2960– 4040	180	4	3320– 4840	165	4
3380– 4150	240	4	2550– 3090	220	4				5730– 6120	190	4			
4150– 4470	290	5	3870– 4390	230	4	5100– 6200	245	5				7340– 8830	200	5
5370– 6610	300	5	4390– 5170	275	5	8180– 9620	255	5	6120– 7290	220	5	12800– 16600	210	5
			6470– 7390	285	5	9620– 12900	295	6	10100– 12700	230	5	16600– 22600	240	6
6610– 6940	350	6							12700– 17700	265	6			
8350– 10400	360	6	7390– 8490	330	6	17500– 20400	340	7				35700– 40500	275	7
10400– 12800	415	7	10600– 12300	340	6	26900– 32300	350	7	25200– 31000	305	7	58200– 79100	285	7
16100– 19400	475	8	12300– 13800	385	7	32300– 41800	390	8	43200– 51500	315	7	79100–102000	315	8
			17200– 20200	395	7				51500– 74000	350	8	151000–172000	325	8
24800– 29300	535	9				58000– 65400	435	9						
36000– 38100	545	9	20200– 22100	440	8	85700–106000	445	9	101000–128000	390	9	172000–180000	350	9
38100– 43900	595	10	27600– 33000	450	8	106000–132000	485	10	179000–200000	400	9			
53900– 58200	605	10	33000– 35400	495	9	177000–190000	495	10						
			44000– 53600	505	9									
58200– 65600	655	11				190000–200000	530	11						
80400– 88700	665	11	53600– 56300	550	10									
88700– 97700	715	12	69800– 87000	560	10									
119000–135000	725	12	87000–110000	610	11									
			140000–174000	665	12									
135000–145000	775	13												
177000–200000	785	13												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.70\,\%$

$p_2 = 1.50\,\%$			$p_2 = 1.75\,\%$			$p_2 = 2.00\,\%$			$p_2 = 2.50\,\%$			$p_2 = 3.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 21600	Accept		1– 7440	Accept		1– 3220	Accept		1– 826	Accept		1– 220	Accept	
21600– 22400	800	11	7440– 8330	380	6	3220– 3300	155	3	827– 902	40	1	221– 365	5	0
22400– 23500	890	12	8330– 9140	465	7	3470– 3630	230	4	1050– 1120	100	2			
24300– 25500	985	13	9550– 10100	550	8	4090– 4250	240	4	1290– 1520	110	2	366– 425	45	1
26400– 27700	1080	14	11000– 12600	640	9							503– 612	55	1
						4250– 4470	310	5	1580– 1680	170	3	752– 885	110	2
28800– 30100	1175	15	12600– 13600	725	10	5050– 5220	320	5	1930– 2260	180	3	1050– 1270	120	2
31300– 32800	1270	16	14500– 15000	810	11	5220– 5460	390	6	2260– 2360	240	4			
34200– 35700	1365	17	16500– 18300	900	12	6170– 6380	400	6	2700– 3130	250	4	1270– 1320	170	3
37200– 38800	1460	18	18900– 19900	985	13							1550– 1860	180	3
40600– 42200	1555	19	21600– 24600	1075	14	6380– 6600	470	7	3130– 3660	315	5	1950– 2130	235	4
						7450– 7760	480	7	4200– 4820	385	6	2520– 2880	245	4
44300– 45900	1650	20	24600– 26400	1160	15	7760– 8930	555	8	5540– 6260	455	7			
48300– 49900	1745	21	28000– 31800	1250	16	9360– 10600	635	9	7230– 8020	525	8	2880– 3290	300	5
52600– 54200	1840	22	31800– 34700	1335	17				9340– 10200	595	9	3950– 4120	310	5
57300– 58900	1935	23	36100– 37500	1420	18	11200– 12600	715	10				4120– 5000	365	6
62500– 63900	2030	24	41000– 45400	1510	19	13400– 14800	795	11	12000– 12800	665	10	5750– 6280	425	7
						16000– 17300	875	12	15300– 16100	735	11	7500– 7970	435	7
68000– 69300	2125	25	46400– 48900	1595	20	19000– 20200	955	13	18800– 19500	745	11			
74100– 75200	2220	26	52600– 59400	1685	21	22400– 23600	1035	14				7970– 9250	490	8
80700– 87800	2320	27	59400– 63600	1770	22				19500– 23400	810	12	10900– 11400	550	9
87800– 95500	2415	28	67300– 75900	1860	23	26500– 27400	1115	15	24700– 29200	880	13	13600– 14900	560	9
95500–104000	2510	29	75900– 82500	1945	24	31300– 36800	1200	16	31200– 36200	950	14	14900– 16600	615	10
						36800– 41900	1280	17				20200– 24000	680	11
104000–113000	2605	30	85700– 88500	2030	25	43300– 48400	1360	18	39400– 44800	1020	15			
113000–123000	2700	31	96700–107000	2120	26	50800– 55800	1440	19	49500– 55400	1090	16	27200– 29100	740	12
123000–133000	2795	32	109000–114000	2205	27				62200– 68300	1160	17	34900– 36700	750	12
133000–145000	2890	33	123000–138000	2295	28	59500– 64400	1520	20	78000– 84200	1230	18	36700– 41800	805	13
145000–158000	2985	34	138000–147000	2380	29	69700– 74200	1600	21	97700–104000	1300	19	49400– 60300	870	14
						81600– 85400	1680	22				66100– 72200	930	15
158000–171000	3080	35	156000–176000	2470	30	95400– 98200	1760	23	122000–127000	1370	20			
171000–186000	3175	36	176000–190000	2555	31	112000–130000	1845	24	153000–183000	1445	21	88300–103000	995	16
186000–200000	3270	37	198000–200000	2640	32				192000–200000	1515	22	118000–124000	1055	17
						130000–148000	1925	25				149000–158000	1065	17
						152000–169000	2005	26				158000–176000	1120	18
						177000–194000	2085	27						

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Single Sampling Tables for $p_1 = 0.70\,\%$

$p_2 = 3.50\,\%$			$p_2 = 4.00\,\%$			$p_2 = 5.00\,\%$			$p_2 = 6.00\,\%$			$p_2 = 7.00\,\%$		
N	n	c	N	n	c	N	n	c	N	n	c	N	n	c
1– 80	Accept		1– 42	Accept		1– 17	Accept		1– 8	Accept		1– 5	Reject	
81– 122	5	0	43– 68	5	0	18– 33	5	0	9– 21	5	0	6– 16	5	0
190– 272	15	0	105– 162	15	0	56– 87	15	0	40– 65	15	0	33– 55	15	0
						140– 194	25	0	107– 172	25	0	95– 160	25	0
273– 324	50	1	233– 256	50	1									
393– 489	60	1	314– 394	60	1	195– 209	50	1	173– 205	50	1	161– 223	50	1
634– 746	110	2	520– 572	70	1	267– 351	60	1	277– 394	60	1	326– 455	60	1
907– 1140	120	2				505– 595	100	2	480– 649	95	2	456– 520	85	2
			573– 613	105	2	771– 1020	110	2	914– 1030	105	2	738– 1050	95	2
1140– 1190	165	3	748– 945	115	2									
1420– 1760	175	3	1080– 1210	160	3	1020– 1120	145	3	1030– 1330	135	3	1050– 1490	125	3
1850– 2090	225	4	1490– 1830	170	3	1450– 1900	155	3	1870– 2040	145	3	2220– 2890	160	4
2540– 2880	235	4				1900– 2570	195	4	2040– 2570	175	4	4560– 5490	195	5
			1830– 2220	215	4	3360– 4420	240	5	3620– 3910	185	4	8080– 9600	205	5
2880– 3550	285	5	2810– 2960	225	4									
4310– 4860	340	6	2960– 3210	265	5	5820– 7450	285	6	3910– 4840	215	5	9600– 15000	235	6
5970– 6400	350	6	3980– 4700	275	5	9940– 12400	330	7	6830– 7390	225	5	19300– 27600	270	7
6400– 7990	400	7				16800– 20600	375	8	7390– 9000	255	6	38400– 50700	305	8
			4700– 5570	320	6	28400– 33900	420	9	12700– 13800	265	6			
9320– 10600	455	8	7260– 7750	370	7							76000– 93000	340	9
13500– 14100	510	9	9670– 11300	380	7	47500– 55700	465	10	13800– 16600	295	7	138000–152000	350	9
17200– 19500	520	9				74500– 80000	475	10	23400– 25600	305	7	152000–170000	375	10
19500– 22600	570	10	11300– 13200	425	8	80000– 91000	510	11	25600– 30300	335	8			
			17200– 22600	480	9	121000–134000	520	11	42700– 47100	345	8			
27900– 29700	625	11	26200– 30500	530	10									
36400– 40100	635	11				134000–148000	555	12	47100– 55200	375	9			
40100– 47300	685	12	39700– 51700	585	11				77700– 86500	385	9			
57100– 61600	740	13	59900– 69200	635	12				86500–100000	415	10			
75900– 81400	750	13	90200–117000	690	13				140000–158000	425	10			
			136000–155000	740	14									
81400– 97800	800	14							158000–181000	455	11			
115000–127000	855	15												
157000–164000	865	15												
164000–200000	915	16												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use $(n+5,c)$ as optimum plan.

Relation between p_r and w_2 for fixed $(p_{10}, p_{20}, \gamma_2)$.

Use the same sampling plan for $w_2 = 0.05$ and $p_{r0} = 0.01$ (0.10) as for w_2 and $p_r = 0.01f$ $(0.10f)$ where f is given in the table.

$100w_2$	$\varrho_2 = p_{20}/p_{r0}$ 1.5	2.0	3.0	5.0	7.0	$\varrho_1 = p_{10}/p_{r0}$
1	0.51	0.44	0.40	0.38	0.37	0.2
	0.78	0.77	0.76	0.76	0.76	0.7
2	0.70	0.63	0.58	0.55	0.53	0.2
	0.85	0.84	0.83	0.82	0.82	0.7
3	0.83	0.78	0.73	0.70	0.69	0.2
	0.91	0.89	0.89	0.88	0.88	0.7
4	0.93	0.90	0.87	0.86	0.85	0.2
	0.96	0.95	0.94	0.94	0.94	0.7
5	1.00	1.00	1.00	1.00	1.00	0.2
	1.00	1.00	1.00	1.00	1.00	0.7
6	1.06	1.09	1.11	1.14	1.15	0.2
	1.04	1.05	1.05	1.06	1.06	0.7
7	1.10	1.16	1.22	1.27	1.29	0.2
	1.07	1.09	1.11	1.12	1.12	0.7
8	1.14	1.22	1.31	1.39	1.43	0.2
	1.10	1.13	1.16	1.17	1.18	0.7
9	1.18	1.28	1.40	1.51	1.56	0.2
	1.12	1.17	1.21	1.23	1.24	0.7
10	1.20	1.33	1.48	1.63	1.69	0.2
	1.15	1.20	1.25	1.29	1.30	0.7
12	1.25	1.41	1.63	1.84	1.95	0.2
	1.19	1.27	1.34	1.40	1.42	0.7
14	1.28	1.48	1.75	2.03	2.19	0.2
	1.22	1.33	1.43	1.51	1.54	0.7
16	1.31	1.54	1.86	2.22	2.41	0.2
	1.25	1.38	1.51	1.62	1.67	0.7
18	1.33	1.58	1.95	2.38	2.63	0.2
	1.27	1.42	1.59	1.72	1.79	0.7
20	1.35	1.63	2.03	2.54	2.84	0.2
	1.29	1.46	1.66	1.83	1.91	0.7

Table of b_1, b_2, and b_3.

$\varrho_1 = p_1/p_r$	$\varrho_2 = p_2/p_r$ 1.5	2.0	3.0	5.0	7.0	
	0.63	0.64	0.65	0.66	0.67	b_1
0.2	0.24	0.27	0.31	0.35	0.37	b_2
	1.42	1.29	1.15	1.03	0.96	b_3
	0.61	0.62	0.64	0.65	0.65	b_1
0.3	0.19	0.23	0.27	0.31	0.34	b_2
	1.69	1.48	1.28	1.11	1.03	b_3
	0.60	0.61	0.63	0.64	0.64	b_1
0.4	0.16	0.19	0.24	0.29	0.32	b_2
	1.99	1.69	1.41	1.19	1.08	b_3
	0.59	0.60	0.62	0.63	0.63	b_1
0.5	0.13	0.17	0.21	0.27	0.30	b_2
	2.33	1.91	1.54	1.27	1.14	b_3
	0.58	0.59	0.61	0.62	0.62	b_1
0.6	0.11	0.15	0.19	0.25	0.28	b_2
	2.74	2.15	1.68	1.34	1.19	b_3
	0.58	0.59	0.60	0.62	0.62	b_1
0.7	0.09	0.13	0.18	0.23	0.26	b_2
	3.24	2.42	1.82	1.42	1.25	b_3

Conversion factor f_2 for N due to a change in $p_r = p_s$ for fixed (p_1, p_2, w_2).
Use $N^* = Nf_2$ as argument in the master table to find (n^*, c^*).
$p_r = p_s = 0.01\lambda$ or 0.10λ, (p_1, p_2, w_2) are given in the master tables, $\varrho_1 = 100p_1$ or $10p_1$, $\varrho_2 = 100p_2$ or $10p_2$.

ϱ_2	ϱ_1	$\lambda = 0.50$	0.60	0.70	0.80	0.90	1.00	1.25	1.50	1.75	2.00	3.00
1.5	0.2	1.80	1.52	1.33	1.18	1.08	1.00	–	–	–	–	–
	0.3	2.21	1.79	1.50	1.28	1.12	1.00	–	–	–	–	–
	0.4	–	2.11	1.69	1.38	1.16	1.00	–	–	–	–	–
	0.5	–	–	1.90	1.50	1.21	1.00	–	–	–	–	–
	0.6	–	–	–	1.64	1.27	1.00	–	–	–	–	–
	0.7	–	–	–	–	–	1.00	–	–	–	–	–
2.0	0.2	1.69	1.47	1.30	1.18	1.08	1.00	0.87	–	–	–	–
	0.3	1.92	1.63	1.41	1.24	1.10	1.00	0.82	–	–	–	–
	0.4	–	1.79	1.52	1.30	1.13	1.00	0.78	–	–	–	–
	0.5	–	–	1.62	1.36	1.16	1.00	0.73	0.58	–	–	–
	0.6	–	–	–	1.42	1.19	1.00	0.69	0.52	–	–	–
	0.7	–	–	–	–	1.21	1.00	0.65	0.47	–	–	–
3.0	0.2	1.53	1.38	1.25	1.15	1.07	1.00	0.88	0.80	–	–	–
	0.3	1.64	1.46	1.31	1.19	1.08	1.00	0.85	0.74	0.67	–	–
	0.4	–	1.52	1.36	1.22	1.10	1.00	0.82	0.70	0.62	0.56	–
	0.5	–	–	–	1.24	1.11	1.00	0.80	0.66	0.57	0.50	–
	0.6	–	–	–	–	1.12	1.00	0.78	0.63	0.53	0.46	–
	0.7	–	–	–	–	–	1.00	0.76	0.60	0.49	0.41	–
5.0	0.2	1.39	1.28	1.19	1.12	1.05	1.00	0.89	0.82	0.76	0.72	–
	0.3	–	1.31	1.22	1.13	1.06	1.00	0.88	0.79	0.72	0.67	–
	0.4	–	–	1.23	1.14	1.07	1.00	0.86	0.76	0.69	0.63	0.50
	0.5	–	–	–	1.14	1.07	1.00	0.85	0.74	0.66	0.60	0.46
	0.6	–	–	–	–	–	1.00	0.85	0.73	0.65	0.58	0.42
	0.7	–	–	–	–	–	1.00	0.85	0.73	0.63	0.56	0.39
7.0	0.2	1.31	1.23	1.16	1.10	1.05	1.00	0.91	0.84	0.79	0.74	0.65
	0.3	–	1.24	1.17	1.11	1.05	1.00	0.89	0.81	0.75	0.70	0.59
	0.4	–	–	–	1.11	1.05	1.00	0.89	0.80	0.74	0.68	0.55
	0.5	–	–	–	–	1.05	1.00	0.89	0.79	0.72	0.66	0.52
	0.6	–	–	–	–	–	1.00	0.89	0.79	0.72	0.65	0.50
	0.7	–	–	–	–	–	1.00	0.90	0.80	0.72	0.65	0.48

Correction g_2 to n^* due to a change in $p_r = p_s$.
Reference value $p_r = p_s = 0.010$. $n = n^* + g_2$.

ϱ_2	ϱ_1	$\lambda = 0.50$	0.60	0.70	0.80	0.90	1.00	1.25	1.50	1.75	2.00	3.00
1.5	0.2	130	100	70	50	25	0	–	–	–	–	–
	0.3	160	120	85	55	30	0	–	–	–	–	–
	0.4	–	150	105	65	35	0	–	–	–	–	–
	0.5	–	–	135	85	40	0	–	–	–	–	–
	0.6	–	–	–	115	50	0	–	–	–	–	–
	0.7	–	–	–	–	–	0	–	–	–	–	–
2.0	0.2	75	55	40	25	15	0	−30	–	–	–	–
	0.3	95	70	50	30	15	0	−35	–	–	–	–
	0.4	–	90	60	35	15	0	−40	–	–	–	–
	0.5	–	–	80	45	20	0	−45	−90	–	–	–
	0.6	–	–	–	60	25	0	−55	−105	–	–	–
	0.7	–	–	–	–	40	0	−70	−125	–	–	–
3.0	0.2	40	30	20	15	5	0	−15	−25	–	–	–
	0.3	55	40	25	15	5	0	−15	−30	−45	–	–
	0.4	–	50	30	20	10	0	−20	−35	−50	−65	–
	0.5	–	–	–	25	10	0	−20	−40	−55	−70	–
	0.6	–	–	–	–	15	0	−25	−45	−60	−80	–
	0.7	–	–	–	–	–	0	−30	−55	−75	−90	–
5.0	0.2	20	15	10	5	5	0	−5	−15	−20	−20	–
	0.3	–	20	15	10	5	0	−10	−15	−20	−25	–
	0.4	–	–	15	10	5	0	−10	−15	−20	−25	−45
	0.5	–	–	–	10	5	0	−10	−20	−25	−30	−50
	0.6	–	–	–	–	–	0	−10	−20	−30	−35	−55
	0.7	–	–	–	–	–	0	−15	−25	−35	−40	−60
7.0	0.2	15	10	5	5	0	0	−5	−10	−10	−15	−25
	0.3	–	15	10	5	0	0	−5	−10	−10	−15	−25
	0.4	–	–	–	5	5	0	−5	−10	−15	−15	−25
	0.5	–	–	–	–	5	0	−5	−10	−15	−20	−30
	0.6	–	–	–	–	–	0	−10	−15	−20	−20	−35
	0.7	–	–	–	–	–	0	−10	−15	−20	−25	−35

For $p_r = p_s = 0.10$ the correction is $g_2/10$ (rounded down).

Conversion factor f_1 for N due to a change in w_2.
Reference value of $w_2 = 0.05$, $p_s = p_r$.
Use $N^* = Nf_1$ as argument in the master table to find (n^*, c^*).

$100w_2$	p_2/p_r 1.5	2.0	3.0	5.0	7.0	p_1/p_r
1	0.54	0.56	0.58	0.61	0.63	0.2
	0.46	0.48	0.51	0.54	0.57	0.7
2	0.70	0.72	0.74	0.76	0.77	0.2
	0.65	0.66	0.68	0.71	0.73	0.7
3	0.82	0.83	0.84	0.86	0.87	0.2
	0.78	0.79	0.81	0.83	0.84	0.7
4	0.92	0.92	0.93	0.94	0.94	0.2
	0.90	0.90	0.91	0.92	0.93	0.7
5	1.00	1.00	1.00	1.00	1.00	0.2
	1.00	1.00	1.00	1.00	1.00	0.7
6	1.07	1.07	1.06	1.06	1.05	0.2
	1.09	1.09	1.08	1.07	1.06	0.7
7	1.14	1.13	1.12	1.10	1.09	0.2
	1.17	1.16	1.15	1.13	1.12	0.7
8	1.19	1.18	1.16	1.15	1.13	0.2
	1.25	1.24	1.21	1.19	1.17	0.7
9	1.25	1.23	1.21	1.18	1.17	0.2
	1.32	1.30	1.27	1.24	1.22	0.7
10	1.30	1.27	1.25	1.22	1.20	0.2
	1.39	1.37	1.33	1.29	1.26	0.7
12	1.38	1.36	1.32	1.28	1.25	0.2
	1.51	1.48	1.43	1.38	1.34	0.7
14	1.46	1.43	1.38	1.33	1.30	0.2
	1.63	1.58	1.52	1.45	1.40	0.7
16	1.53	1.49	1.44	1.38	1.34	0.2
	1.73	1.68	1.61	1.52	1.46	0.7
18	1.60	1.55	1.49	1.42	1.38	0.2
	1.83	1.77	1.68	1.58	1.52	0.7
20	1.65	1.60	1.53	1.46	1.41	0.2
	1.92	1.85	1.75	1.64	1.56	0.7

Correction g_1 to n^* due to a change in w_2.
Reference value of $w_2 = 0.05$, $p_s = p_r = 0.01$. $n = n^* + g_1$.

$100w_2$	p_2/p_r 1.5	2.0	3.0	5.0	7.0	p_1/p_r
1	−125	−90	−60	−35	−25	0.2
	−205	−125	−70	−35	−25	0.7
2	−70	−50	−35	−20	−15	0.2
	−115	−70	−40	−20	−15	0.7
3	−40	−30	−20	−10	−10	0.2
	−65	−40	−25	−10	−10	0.7
4	−20	−15	−10	−5	−5	0.2
	−30	−20	−10	−5	−5	0.7
5	0	0	0	0	0	0.2
	0	0	0	0	0	0.7
6	15	10	5	5	5	0.2
	25	15	10	5	5	0.7
7	25	20	15	5	5	0.2
	45	25	15	10	5	0.7
8	40	30	20	10	5	0.2
	60	40	20	10	10	0.7
9	50	35	20	15	10	0.2
	80	50	25	15	10	0.7
10	55	40	25	15	10	0.2
	90	55	30	15	10	0.7
12	75	50	35	20	15	0.2
	120	70	40	20	15	0.7
14	85	60	40	25	15	0.2
	140	85	50	25	15	0.7
16	100	70	45	25	20	0.2
	160	100	55	30	20	0.7
18	110	80	50	30	20	0.2
	175	110	60	30	20	0.7
20	120	85	55	30	20	0.2
	195	120	65	35	25	0.7

For $p_s = p_r = 0.10$ the correction is $g_1/10$ (rounded down).

Summary of conversion formulas
to find
(n, c) corresponding to $(N, p_r, p_s, p_1, p_2, w_2)$
from
(n^*, c^*) in the master table for $(N^*, p_{r0}, p_{10}, p_{20})$.

$$\text{For} \begin{cases} p_r \leqq 0.05 \\ p_r > 0.05 \end{cases} \text{use master table with } p_{r0} = \begin{cases} 0.01 \\ 0.10 \end{cases}$$

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)^{-1}.$$

Formula 1.

$$\gamma_2 = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)} \quad \text{and} \quad \lambda p_{r0} = \frac{p_2 + 19\gamma_2 p_1}{1 + 19\gamma_2}.$$

Use

$$N^* = N\lambda_s\lambda, \quad p_{r0}, \quad p_{10} = p_1/\lambda, \quad p_{20} = p_2/\lambda$$

as arguments to find (n^*, c^*) in the master table.

$$(n, c) = (n^*/\lambda, c^*).$$

If (p_{10}, p_{20}) fall outside the tabulated range use formula 2.

Formula 2.

$$\lambda = p_r/p_{r0}, \quad \varrho_1 = p_1/p_r, \quad \varrho_2 = p_2/p_r.$$

Use

$$N^* = N\lambda_s\lambda f_1(w_2, \varrho_1, \varrho_2), \quad p_{r0}, \quad p_{10} = \varrho_1 p_{r0}, \quad p_{20} = \varrho_2 p_{r0}$$

as arguments to find (n^*, c^*) in the master table.

$$(n, c) = \left((n^* + g_1(w_2, \varrho_1, \varrho_2))/\lambda, c^*\right).$$

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